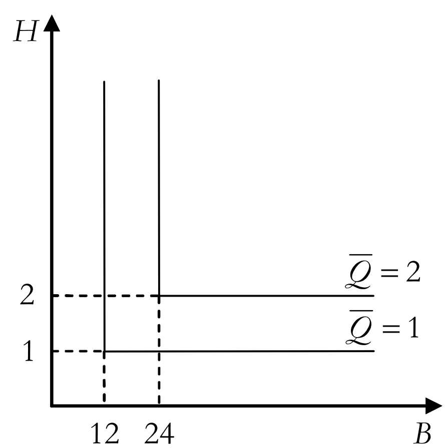
ECON 3070
HW 4
Suggested solutions

March 19, 2009

Problem 1 *Coors makes beer with hops and barley . For every keg of beer, Coors needs exactly 12 ounces of barley for every 1 ounce of hops.*

1. Draw the isoquants for beer production.

   Solution 1 *12 ounces of barley and 1 ounce of hops are perfect complements.*



2. Write a mathematical expression for the production function for beer.

Solution 2

$$Q = F(B,H) = \min\left\{\frac{B}{12}, H\right\}$$

For example, if $B = 12$ and $H = 1$ then the output, Q, in kegs, is equal to unity.

3. Suppose barley costs \$0.70 an ounce and hops costs \$2.00 an ounce. What is the (smallest) cost of producing 1 keg of beer? 2 kegs? $Q > 0$ kegs?

Solution 3 *Since Coors uses 12 ounces of barley and 1 ounce of hops to make 1 keg, it costs Coors $12 * .7 + 1 * 2 = 10.4$ to produce each keg. For Q kegs then the cost is $Q(12 * \$.7 + 1 * \$2) = \$10.4 * Q$.*

Problem 2 *A firm with production function $F(L,K) = L^{2/3}K^{1/3}$, facing input prices $w = 10, r = 5$ hires you to provide answers to accomplish the following tasks.*

1. *Find the optimal input combination to produce $Q > 0$ units of output.*

Solution 4

$$\begin{aligned} & \min_{L,K} 10L + 5K \\ & s.t. \\ L^{2/3}K^{1/3} & \geq Q \end{aligned}$$

Optimality conditions:

$$\begin{aligned} \frac{2K^*}{L^*} &= MRTS_{L,K}(L^*,K^*) = \frac{10}{5} \\ L^{*2/3}K^{*1/3} &= Q \end{aligned}$$

imply

$$\begin{aligned} L^* &= K^*, \\ L^{*2/3}L^{*1/3} &= L^* = Q \end{aligned}$$

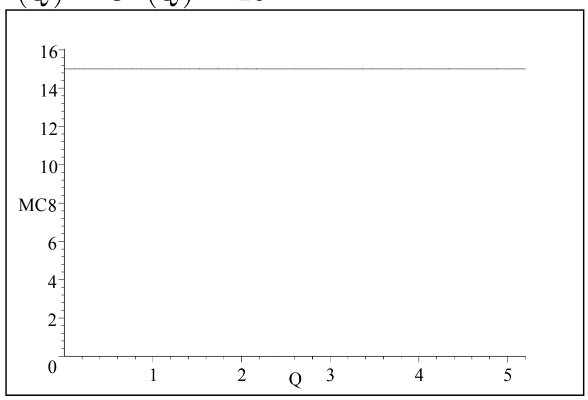
So, optimal combination is $L^* = K^* = Q$.

2. *Find the cost function,* $C(Q)$.

   Solution 5 $C(Q) = 10L^* + 5K^* = 15Q$

3. *Calculate and plot marginal cost.*

   Solution 6 $MC(Q) = C'(Q) = 15$



4. *Does the production exhibit increasing, decreasing or constant returns to scale? (Is your finding consistent with the shape of the marginal cost you found in the previous question?)*

   Solution 7 *Production exhibits constant returns to scale, and MC is constant as well.*

Problem 3 *Firm Jimba produces candy and its variable cost (in thousand dollars) is* $VC(q) = 2q^3$*, where* q *is a ton of candy. Assume, in addition, there is a fixed (sunk) cost of one thousand dollars,* $FC = 1$.

1. *What is the optimal output for this firm if the market price for a ton is* $P = 96$ *thousand dollars?*

   Solution 8 *The goal of the firm is to maximize its profits,* $\pi(q) = Pq - (FC + VC(q))$ *Therefore a price-taking firm is choosing a quantity of output such that its profit is maximized:*

$$\max_q Pq - (1 + 2q^3)$$

First order conditions require $\frac{d\pi(q)}{dq} = 0$, *or*

$$P - 6q^2 = 0 \tag{1}$$

as $P = 96$, *it implies*

$$\begin{aligned} 16 &= q^2 \\ q &\in \{4, -4\} \end{aligned}$$

Note that a firm can not produce a negative quantity of output (you can think of this as just a costly way of "wasting" ouput), so the only candidate solution is $q = 4$. *We need to check that it, indeed, maximizes the profit, thus verify the second order conditions,* $\frac{d^2\pi(q)}{(dq)^2} < 0$, *in this problem*

$$\frac{d^2\pi(4)}{(dq)^2} = -12 * 4 = 48 < 0$$

So, $q = 4$ *does maximize the profit of the firm.*

2. *At what level of fixed cost will the firm earn zero economic profit?*

Solution 9 *Note that fixed cost does not affect the choice of optimal quantity of output, i.e., no matter how high is the fixed cost the first order condition* (1) *does not change. (Fixed cost affects the shutdown decisions only.) So, the firm should choose* $q = 4$. *Then its profit is*

$$\begin{aligned} \pi(4) &= 4P - (FC + VC(4)) = \\ &= 384 - FC - 2\left(4^3\right) = \\ &= 256 - FC \end{aligned}$$

to make it equal to zero, the fixed cost has to be 256.