ECON 3070
INTERMEDIATE MICROECONOMIC THEORY
Spring 2009, Homework 1

Suggested solutions

Carefully show how you derive you answer and be sure to interpret your answer where necessary.

1. Consider a market with (aggregate) demand given by

$$Q^d(p) = a - bp$$

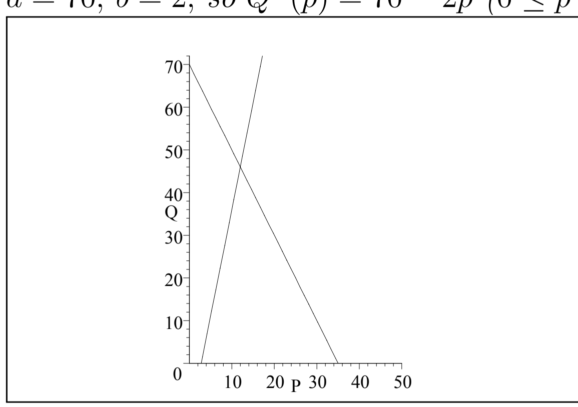
where a, b are positive constants and p is the price of the good. Supply on that market is given by

$$Q^s(p) = c + zp$$

where, again p is the price and c, z are some constants.

(a) Pick constants a, b and graph the demand.

Solution 1 *Let* $a = 70$, $b = 2$, *so* $Q^d(p) = 70 - 2p$ $(0 \leq p \leq 35)$



(b) Pick constants c, z and depict the supply on the previous graph.

Solution 2 *Let* $c = -14$, $d = 5$, *so* $Q^s(p) = -14 + 5p$ *(for* $p > \frac{14}{5}$*).*

(c) Is there an equilibrium? If so, what is the quantity demanded and supplied and what is the equilibrium price? Demonstrate your answer both algebraically (by solving for equilibrium price and quantity) and graphically (by showing the point corresponding to the equilibrium).

Solution 3 $Q^d(p) = 70 - 2p$ *and* $Q^s(p) = -14 + 5p$. *Set* $Q^d(p^*) = Q^s(p^*)$ *to get* $70 - 2p^* = -14 + 5p^*$, *Solution is:* $p^* = 12$, *so* $Q^* = 46$; *this is the equilibrium.*

(d) Calculate elasticity of demand at the equilibrium point.

Solution 4 *Price elasticity of demand is* $\varepsilon^d(p^*) = Q^{d\prime}(p^*)\frac{p^*}{Q^*} = -2 * \frac{12}{46} = -\frac{12}{23}$.

(e) Now increase constant a in the equation of demand (that could happen if some potential buyers became richer), so that the new demand is

$$\hat{Q}^d(p) = \hat{a} - bp$$

where $\hat{a} = 1.1a$. How does the equilibrium price change as a result of this decrease? Will more or less of the good be sold on that market?

Solution 5 *New demand now is* $\hat{Q}^d(p) = 70 * 1.1 - 2p = 77.0 - 2p$. *Supply is the same as before, so equating the two we get* $77.0 - 2p^* = -14 + 5p^*$, *Solution is:* $p = 13.0$, *so quantity sold now is* $-14 + 5 * 13 = 51$, *which is higher than before the change.*

2. A monopolist faces direct demand of $Q^d(p) = 1000 - 20p$, where $Q^d(p)$ is quantity demanded at price p.

(a) What is the inverse demand function?

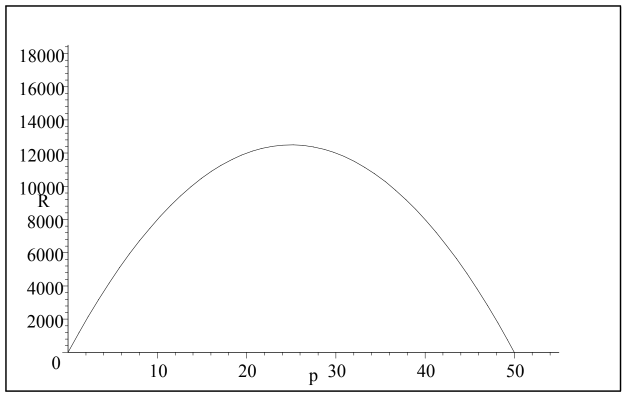
Solution 6 *The inverse demand is* $p(Q) = 50 - \frac{1}{20}Q$.

(b) What is the monopolist's total revenue function?

Solution 7 $R(p) = Q^d(p)\,p = (1000 - 20p)p = 1000p - 20p^2$

(c) Graph the revenue function, $R(p) = Q^d(p)\,p$.

Solution 8



(d) Find the monopolist's choice of price that maximizes total revenue. Use both first and second order conditions. What is the highest revenue the monopolist can get?

Solution 9 *The problem is*

$$\max_p 1000p - 20p^2$$

F.O.C.: $R'(p^*) = 0$, *implying* $1000 - 40p^* = 0$, *so* $p^* = 25$.
S.O.C. $R''(p^*) < 0$ *are satisfied, as* $R''(p^*) = -40 < 0$.

(e) What is the elasticity of demand at that point?
$\varepsilon^d(p^*) = Q^{d\prime}(p^*)\frac{p^*}{Q^*(p^*)} = -20\frac{25}{1000-20*25} = -1.$

(f) What is the revenue at the point where the demand is unit-elastic?

Solution 10 *By the previous calculation, it is the point where the revenue is maximized:* $R(25) = 1000 * 25 - 20(25)^2 = 12\,500.$