Practice Problems with Answers for Exam 3
Econ 3070-002
Spring 2004

Name____

1. You have an income of \$$m$ to spend on two commodities, x_1 and x_2. x_1 costs \$3 per unit and x_2 costs \$2 per unit. Your utility function is

$$U(x_1, x_2) = \frac{1}{3}\ln(x_1) + \frac{2}{3}\ln(x_2)$$

   (a) Calculate the general form of x_1^* and x_2^*

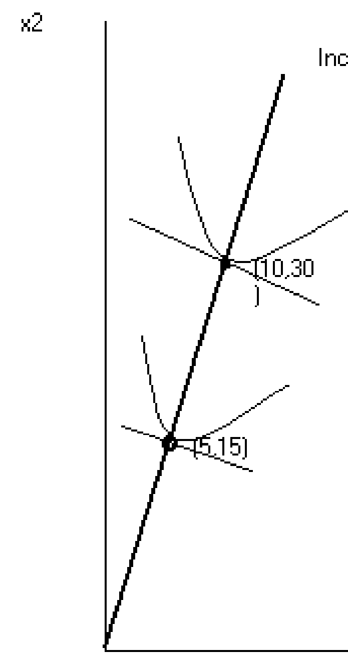
$$\begin{aligned} x_1, x_2 Max \frac{1}{3}\ln(x_1) + \frac{2}{3}\ln(x_2) \text{ s.t. } 3x_1 + 2x_2 &= m \\ & x_1 Max \frac{1}{3}\ln(x_1) + \frac{2}{3}\ln(\frac{m - 3x_1}{2}) \\ \frac{1}{3x_1} + \frac{2}{3\left(\frac{m-3x_1}{2}\right)}\left(\frac{-3}{2}\right) &= 0 \\ x_1^* &= \frac{m}{9} \\ x_2^* &= \frac{m}{3} \end{aligned}$$

   (b) Suppose $m = 45$. Calculate x_1^* and x_2^*.

$$\begin{aligned} x_1^* &= \frac{45}{9} = 5 \\ x_2^* &= \frac{45}{3} = 15 \end{aligned}$$

   (c) Interpret x_1^* and x_2^*.

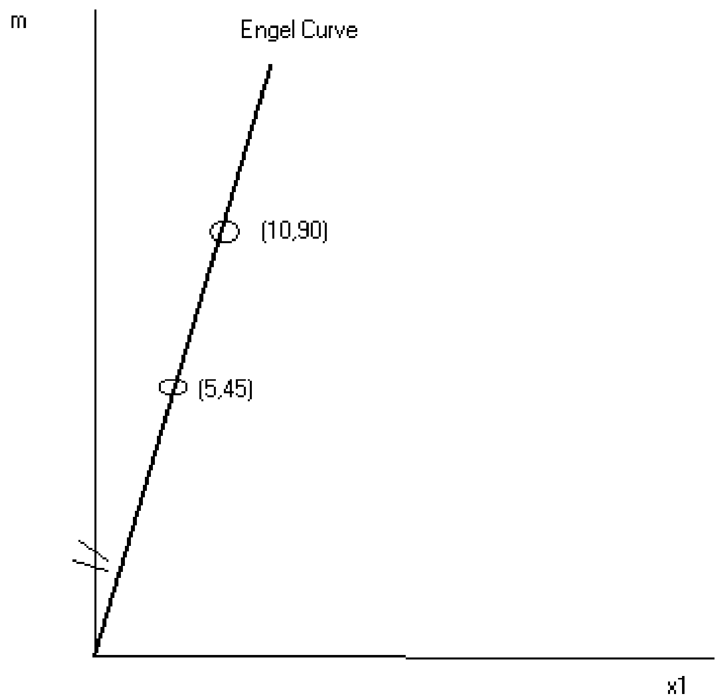
x_1^* and x_2^* represent the optimal amount of x_1 and x_2 that I should buy so as to maximize my utility, given market prices and my income. In this case, I should buy 5 units of x_1 and 15 units of x_2.



2. (a) Sketch the income offer curve, plotting the points associated with $m = 45$ and $m = 90$.

(b) Sketch the Engel curve, plotting the points associated with $m = 45$ and $m = 90$. Identify the slope.

$$\begin{aligned} m &= 9x_1 \\ slope &= 9 \end{aligned}$$



3. You have an income of \$50 to spend on two commodities, x_1 and x_2. x_1 costs \$1 per unit and x_2 costs \$2 per unit. Your utility function is

$$U(x_1, x_2) = Min\{x_1, 3x_2\}$$

Calculate x_1^* and x_2^*.

$$\begin{aligned} x_1 + 2x_2 &= 50 \\ x_1 &= 3x_2 \end{aligned}$$

Combining,

$$\begin{aligned} 3x_2 + 2x_2 &= 50 \\ x_2^* &= 10 \\ x_1 &= 3x_2 = 30 \end{aligned}$$

4. Consider the utility function $U(x_1, x_2) = x_1 + 4x_2$.

(a) Write the condition under which we know with certainty that an individual will buy only x_2

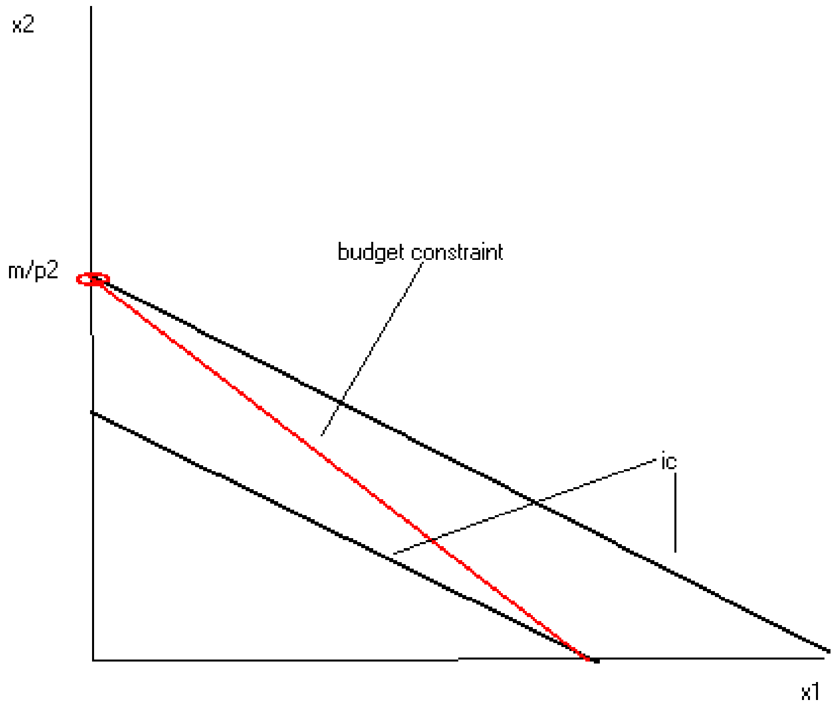
$$\begin{aligned} \text{slope of indifference curve} &= MRS = -\frac{1}{4} \\ x_2 &= \frac{m - p_1 x_2}{p_2} \rightarrow \text{slope of budget constraint}= -\frac{p_1}{p_2} \end{aligned}$$

compare slope of budget constraint & indifference curve

must be case that slope of budget constraint steeper than slope of indifference curve

$$\frac{p_1}{p_2} > \frac{1}{4}$$

(b) Draw a graph illustrating this situation. Identify indifference curves, budget constaint, and amount purchased in your graph.



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5. Calculate the price elasticity for the demand curves below. I've written $D(p)$ as the demand function (in class, this was usually q)

$D(p)$	Price Elasticity of Demand
$D(p) = 60 - p$	$-\frac{p}{q}$
$D(p) = a - bp$	$-\frac{bp}{q}$
$D(p) = 40p^2$	
$D(p) = Ap^{-b}$	
$D(p) = (p+3)^{-2}$	
$D(p) = (p+a)^{-b}$	

6. The demand function for football tickets for a typical game at a large Midwestern university is $D(p) = 200,000 - 10,000p$. The university has a clever and avaricious (i.e., greedy, grasping) athletic director who sets his ticket prices so as to maximize revenues. The university's football stadium holds 100,000 spectators.

   (a) Write down the inverse demand function.

   (b) Write expressions for total revenue and marginal revenue as a function of the number of tickets sold.

(c) Use blue to draw the inverse demand function and red to draw the marginal revenue function. Also draw a vertical blue line representing the capacity of the stadium.

(d) What price will generate the maximum revenue? What quantity will be sold at this price?

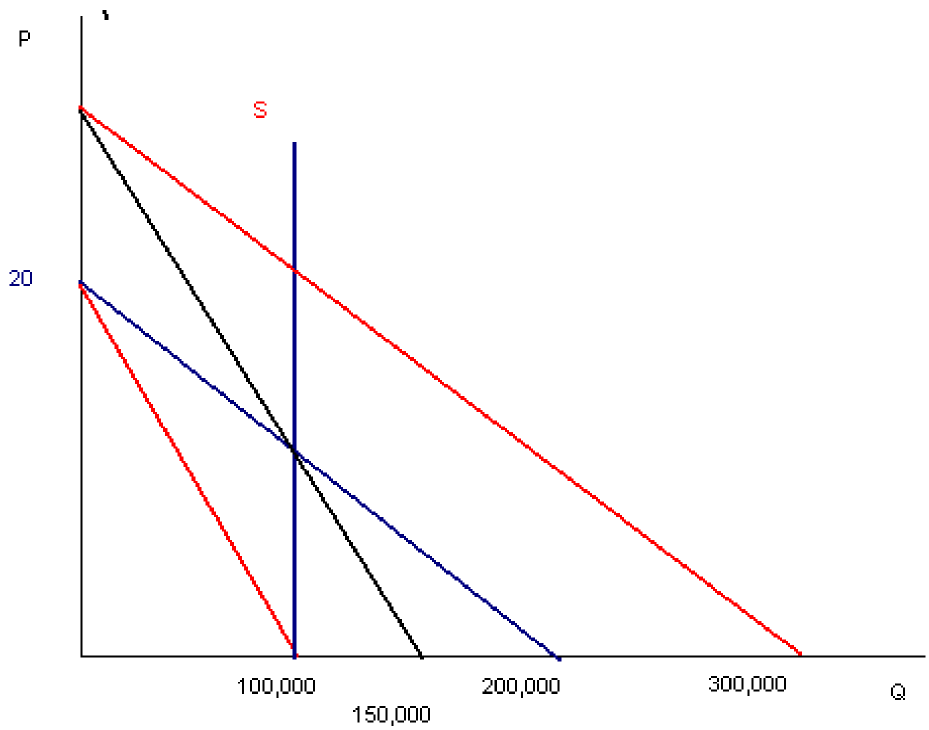
(e) At this quantity, what is the marginal revenue? At this quantity, what is the price elasticity of demand? Will the stadium be full?

ANSWERS:

(a) $P = 20 - \frac{Q}{10{,}000}$

(b)

$$\begin{aligned} \mathrm{Re}\,v &= \left(20 - \frac{Q}{10000}\right) Q \\ MR &= -\frac{Q}{10000} + 20 - \frac{Q}{10000} \end{aligned}$$



(c)

$$\begin{aligned} \mathrm{Re}\,v &= P * (200,000 - 10,000P) \\ MR &= 200000 - 10000P - 10000P = 0 \\ P &= 10 \\ Q &= 100,000 \end{aligned}$$

(d)

$$\begin{aligned} MR &= 20 - \frac{2(100,000)}{10,000} = 0 \\ \varepsilon^d &= -10000 * \frac{P}{Q} \\ &= -\frac{10000(10)}{100,000} = 1 \end{aligned}$$