

Economics 3070 Practice Problems
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Make sure to label axes and indifference curves.

1. Fill in the table below.

$u(x_1, x_2)$	MU_1	MU_2	MRS
$2x_1 + 3x_2$			
$4x_1 + 6x_2$			
$ax_1 + bx_2$			
$2x_1^{1/2} + x_2$			
$\ln x_1 + x_2$			
x_1x_2			
$x_1^ax_2^b$			
$(x_1 + 2)(x_2 + 1)$			
$(x_1 + a)(x_2 + b)$			
$x_1^a + x_2^b$			

2. Charlie's utility function is $U(x_A, x_B) = x_Ax_B$.

- (a) Charlie has 40 apples (x_A) and 5 bananas (x_B). Charlie's utility for the bundle (40, 5) is $U(40, 5) = \text{-----}$.
- (b) The indifference curve through (40, 5) includes all commodity bundles (x_A, x_B) such that $x_Ax_B = \text{-----}$. So the indifference curve through (40, 5) has the equation $x_B = \text{-----}$.
- (c) Graph the indifference curve showing all the bundles that Charlie likes exactly as well as the bundle (40, 5). Plot apples on the x axis.
- (d) Donna offers to give Charlie 15 bananas if he will give her 25 apples. Would Charlie have a bundle that he likes better than (40, 5) if he makes this trade?
- (e) What is the largest number of apples that Donna could demand from Charlie in return for 15 bananas if she expects him to be willing to trade or at least indifferent about trading? (Hint: if Donna gives Charlie 15 bananas, he will have a total of 20 bananas. If he has 20 bananas, how many apples does he need in order to be as well-off as he would without trade?)

3. Recall Shirley Sixpack and Lorraine Quiche from HW1. Shirley thinks a 16-ounce can of beers is just as good as two 8-ounce cans. Lorraine only drinks 8 ounces at a time and hates stale beer, so she thinks a 16-ounce can is not better or worse than an 8-ounce can.

- (a) Write a utility function that represents Shirley's preferences between commodity bundles comprised of 8-ounce cans and 16-ounce cans of beer. Let x_1 stand for the number of 8 ounce cans and x_2 stand for the number of 16 ounce cans.
- (b) Write a utility function that represents Lorraine's preferences.

- (c) Would the utility function $U(x_1, x_2) = 100x_1 + 200x_2$ represent Shirley's preferences? Would the utility function $U(x_1, x_2) = (5x_1 + 10x_2)^2$ represent her preferences? Would the utility function $U(x_1, x_2) = (x_1 + 3x_2)$ represent her preferences?
4. Reconsider Charlie again. Recall that his utility function is $U(x_A, x_B) = x_A x_B$. Suppose that $p_A = 1$, $p_B = 2$, and $m = 40$.
- (a) Write Charlie's budget equation.
- (b) Write Charlie's general optimization problem (what will he optimize subject to what budget constraint?)
- (c) Now write the optimization problem in terms of x_B . Solve for x_B^* . Solve for x_A^* .
- (d) Calculate the utility level at x_A^*, x_B^* .
- (e) On a graph with apples (x_A) on the x axis, plot the budget equation in black. In red, draw the indifference curve for the utility level calculated in (d).
- (f) Calculate $MRS(x_A^*, x_B^*)$. What is the slope of the budget equation?
5. Calculate x^*, y^* for the following cases.
- (a) $U(x, y) = (x + 2)(y + 1)$, where $p_x = p_y = 1$, $m = 11$
- (b) $U(x, y) = 4x^{1/2} + y$, where $p_x = 1, p_y = 2, m = 24$
6. Consider the utility function

$$U = \min\{4x_1, x_2\}$$

- (a) Graph the indifference curves for $U = 4$ and $U = 8$.
- (b) Calculate the general form of x_1^* and x_2^*
- (c) Suppose $p_1 = 2, p_2 = 4, m = 36$. Calculate x_1^* and x_2^*
7. Consider the utility function $U(x_A, x_B) = x_A x_B$, where $p_A = 2, p_B = 1$, and $m = 40$. The price of x_A suddenly falls to \$1. x_A are apples and x_B are bananas.
- (a) How many apples and bananas does Charlie consume before the price change? (Hint: it will be easier for the rest of the problem if you figure out the general solution first.) On a graph (with apples on the x axis), in black draw Charlie's original budget line and label his chosen consumption point as A.
- (b) If after the price change, Charlie's income had changed so that he could exactly afford his old consumption bundle, his new income would have been _____. With this income and new prices, Charlie would consume _____ apples and _____ bananas. Use red to draw the budget line corresponding to this income and these prices. Label the bundle that Charlie would choose at this income and the new prices with the letter B.

- (c) Does the substitution effect of the fall in the price of apples make him buy more or fewer apples? How many more or fewer?
- (d) After the price change, Charlie actually buys _____ apples and _____ bananas. Use blue to draw Charlie's actual budget line after the price change. Put the label C on the bundle that he actually chooses after the price change. Draw 3 horizontal lines on your graph, one from A to the vertical axis, one from B to the vertical axis, and one from C to the vertical axis. Along the vertical axis, label the income effect, the substitution effect, and the total effect on the demand for apples.
- (e) The income effect of the fall in the price of apples on Charlie's demand for apples is the same as an (increase, decrease) _____ in his income of \$ _____. Does the income effect make him consume more or fewer apples? How many more or fewer?
- (f) Does the substitution effect of the fall in the price of apples make Charlie consume more bananas or fewer? How many more or fewer? Does the income effect of the fall in the price of apples make Charlie consume more bananas or fewer? What is the total effect of the change in the price of apples on the demand for bananas?
8. Maude spends all her income on delphiniums and hollyhocks. She thinks that delphiniums and hollyhocks are perfect substitutes: one delphinium is just as good as one hollyhock. Delphiniums cost \$4 a unit and hollyhocks cost \$5 a unit. (Hint: sketch graphs)
- (a) If the price of delphiniums decreases to \$3 a unit, will Maude buy more of them? What part of the change in consumption is due to the income effect and what part is due to the substitution effect?
- (b) If the prices of delphiniums and hollyhocks are respectively $p_d = \$4$ and $p_h = 5$ and if Maude has \$120 to spend, draw her budget line in blue ink. Put hollyhocks on the x axis. Draw the highest indifference curve that she can attain in red ink, and label the point that she chooses as *A*.
- (c) Now let the price of hollyhocks fall to \$3 a unit, while the price of delphiniums does not change. Draw her new budget line in black ink. Draw the highest indifference curve that she can now reach with red ink. Label the point she chooses now as *B*.
- (d) How much would Maude's income have to be after the price of hollyhocks fell, so that she could just exactly afford her old commodity bundle *A*?
- (e) When the price of hollyhocks fell to \$3, what part of the change in Maude's demand was due to the income effect and what part was due to the substitution effect?
9. Suppose that two goods are perfect complements. If the price of one good changes, what part of the change in demand is due to the substitution effect and what part is due to the income effect? Draw a graph illustrating this.

10. Douglas Cornfield's demand function for good x is $x(p_x, p_y, m) = \frac{2m}{5p_x}$. His income is \$1000, $p_x = \$5$, $p_y = \$20$.
- If the price of x falls to \$4, then his demand for x will change from _____ to _____.
 - If his income were to change at the same time so that he could exactly afford his old commodity bundle at $p_x = 4$ and $p_y = 20$, what would his new income be? (Hint: you will first need to calculate his demand for y . You can do this w/o remaximizing.....) What would be his demand for x at this new level of income, at prices $p_x = 4$ and $p_y = 20$?
 - The substitution effect in this case is _____ and the income effect is _____.
 - Use blue to draw Douglas Cornfield's budget line before the price change. Locate the bundle he chooses at these prices on your graph and label this point A. Use black to draw his budget line after the price change. Label his consumption at this point as B. In red, draw a budget line with the new prices but with an income that just allows Douglas to buy his old bundle, A. Find the bundle that he would choose with this budget line and label it C. On your graph, indicate the SE, IE, and TE.
11. For the case of a normal good, draw a graph that illustrates the SE, IE, and TE for an increase in price. Use 3 separate colors.
12. For the case of an inferior but non-Giffen good, draw a graph that illustrates the SE, IE, and TE for an increase in price. Use 3 separate colors.