

## Chapter 8:

### Comparative dynamics in a model with a steady-state

Consider a very straight-forward dynamic model with an endogenous capital stock.

#### Sectors (Activities)

- $X_t$  production of composite good in period  $t$
- $I_t$  production of new capital (investment) in period  $t$
- $K_t$  transforms capital into capital services and future capital

#### Commodities (Markets)

- $p_x$  ( $CX_t$ ) price of  $X$  in period  $t$
- $p_r$  ( $CR_t$ ) rental price of capital in period  $t$
- $p_k$  ( $CK_t$ ) asset price of capital (price of a new capital good) in period  $t$
- $p_l$  ( $CL_t$ ) price of labor in period  $t$

#### Consumers

Infinitely lived representative consumer

- $\delta$  = rate of capital depreciation  
 $\rho$  = rate of time preference (discounting utility)  
 $KE_t$  = capital endowment at the beginning of a period  
 $K_t$  = capital stock for production at time t ( $KE_t + I_t$ )

Conditions for Steady-State Equilibrium:

- (1)  $KE_{t+1} = KE_t \Rightarrow I_t = \delta KE_t$
- (2) rate of interest =  $\rho$ :  $pk_{t+1} = \frac{pj_t}{1 + \rho}$  for all goods  $j = X, I, K$
- (3)  $pr_t = \left[ 1 - \frac{1 - \delta}{1 + \rho} \right] pk_t$  relationship between asset and rental prices
- (4)  $KE_{t+1} = (1 - \delta)K_t = (1 - \delta)(KE_t + I_t)$  (KE: capital endowment)
- $$pk_{t+1}KE_{t+1} = (1 - \delta) \frac{pk_t}{1 + \rho} K_t = (1 - \delta) \frac{(pk_t KE_t + pk_t I_t)}{1 + \rho}$$

	Xt	It	Kt	CONS	
CXt	200			-200	0
CRt	-100		100		0
CKt		40	-400	360	0
CLt	-100	-40		140	0
CKt+1			300	-300	0
	0	0	0	0	

Parameters:  $RHO = 0.2$ ,  $DELTA = 0.1$

Prices:  $CX0=CR0=CL0=1$ :  $CK0=4$   $CK1=CK0/(1+RHO)=1/1.2$

$$CR0 = (1 - (1-DELTA)/(1+RHO)) * CK0 =$$

$$CRO = (1 - 0.9/1.2) * CK0 = (1/4) * CK0$$

$$KEt+1 = (1-DELTA) * (KEt+It) = (1-DELTA) * (360+40) = 360$$

$$CKt+1 * KEt+1 = (1-DELTA) * (CKt * KEt + CRt * It) / (1+RHO) = 300$$

Problem: Suppose we want to represent this infinite-horizon problem as a finite dimension complementarity problem

Approaching the last period the consumer would have no incentive to accumulate capital and would want to run down the capital stock.

- (1) Assume a finite number of periods plus a terminal period.
- (2) Assume an extra dummy agent (God? But don't want to offend anyone)
- (3) Assume that the dummy agent is endowed with an extra good "Heaven"
- (4) Assume that the dummy agent will only sell Heaven in exchange for terminal period capital (does not demand any other good)
- (5) Assume that the representative agent has a demand for heaven
- (6) Use a tax/subsidy on heaven to ensure that the asset/rental price relationship holds on terminal capital (so that the economy is forced onto the steady-state path at terminal time)

Terminal period

	Xt	It	Kt	CONS	DUMMY	
CXt	200			-200	0	0
CRt	-100		100		0	0
CKt		40	-400	360	0	0
CLt	-100	-40		140	0	0
CKt+1			300		-300	0
Heaven				-300	300	0
	0	0	0	0		

SETS T /1\*25/;

PARAMETERS

DELTA

RHO

PV

TERM

RTERM

INITK

R(T)

D(T)

PVUTIL

TLAST(T)

TFIRST(T)

SOLUTION(T,\*)

CONSUME(T)

INVEST(T)

KSTOCK(T);

RHO = 0.2;

DELTA = 0.1;

INITK = 90;

```
TERM = CARD(T);
RTERM = (1/(1+RHO))** (CARD(T) - 1);;
R(T) = (1/(1+RHO))** (ORD(T) - 1);
D(T) = (1-DELTA)** (ORD(T) - 1);
PV = 200*SUM(T, R(T)) + 90*(4*RTERM/(1+RHO));
TLAST(T) = 0;
TLAST('25') = 1;
TFIRST('1') = 1;
```

\$ONTEXT

\$MODEL: BASIC

\$SECTORS:

X(T)

I(T)

K(T)

U

\$COMMODITIES:

CX(T)

CR(T)

CK(T)

CL(T)

CKT  
CU  
HEAVEN

\$CONSUMERS:

CONS  
DUMMY

\$AUXILIARY:

TRANS

\$PROD:K (T)

O:CK (T+1)	Q: (100* (1-DELTA) )	P: (4*R (T+1) )
O:CKT\$TLAST (T)	Q: (100* (1-DELTA) )	P: (4*R (T) / (1+RHO) )
O:CR (T)	Q:100	P: (R (T) )
I:CK (T)	Q:100	P: (4*R (T) )

\$PROD:I (T)

O:CK (T)	Q:10
I:CL (T)	Q:40

\$PROD:X(T) s:1

O: CX(T) Q:200

I: CL(T) Q:100

I: CR(T) Q:100

\$PROD:U s:1 a:2

O: CU Q:PV

I: CX(T) Q:200 P: R(T) a:

I: HEAVEN Q:90 P: (4\*RTERM/(1+RHO)) A: CONS N: TRANS

\$DEMAND:CONS

D: CU Q:PV

E: CL(T) Q:140

E: CK(T) \$TFIRST(T) Q: INITK

\$DEMAND:DUMMY

D: CKT Q:90

E: HEAVEN Q:90

\$CONSTRAINT: TRANS

CR('25') =E= (1 - (1-DELTA)/(1+RHO)) \*CL('25') \*4;

```
$OFFTEXT
```

```
$SYSINCLUDE MPSGESET BASIC
```

```
TRANS.UP = +INF;
```

```
TRANS.LO = -INF;
```

```
CX.L(T) = R(T);
```

```
CL.L(T) = R(T);
```

```
CR.L(T) = R(T);
```

```
CK.L(T) = 4*R(T);
```

```
CKT.L = 4*R('25') / (1+RHO);
```

```
HEAVEN.L = 4*R('25') / (1+RHO);
```

```
TRANS.L = 0;
```

```
*BASIC.ITERLIM = 0;
```

```
$INCLUDE BASIC.GEN
```

```
SOLVE BASIC USING MCP;
```

```
PVUTIL = SUM(T, X.L(T)*R(T)) + (X.L('25')*R('25'))/RHO;
```

```
DISPLAY PVUTIL;
```

```
CONSUME(T) = X.L(T);  
INVEST(T) = I.L(T);  
KSTOCK(T) = K.L(T);  
SOLUTION(T, "X") = X.L(T);  
SOLUTION(T, "I") = I.L(T);  
SOLUTION(T, "K") = K.L(T);
```

```
$LIBINCLUDE XLDUMP SOLUTION SOL2.xls SHEET1!A2
```

```
INITK = 30;
```

```
$INCLUDE BASIC.GEN  
SOLVE BASIC USING MCP;
```

```
PVUTIL = SUM(T, X.L(T)*R(T)) + (X.L('25')*R('25'))/RHO;
```

```
DISPLAY PVUTIL;
```

```
CONSUME(T) = X.L(T);  
INVEST(T) = I.L(T);  
KSTOCK(T) = K.L(T);
```

```
SOLUTION(T, "X") = X.L(T);
SOLUTION(T, "I") = I.L(T);
SOLUTION(T, "K") = K.L(T);
```

```
$LIBINCLUDE XLDUMP SOLUTION SOL2.xls SHEET1!F2
```

```
* make people more patient, raise rho to 0.1
```

```
INITK = 90;
RHO = 0.1;
```

```
RTERM = (1/(1+RHO))** (CARD(T) - 1);;
R(T) = (1/(1+RHO))** (ORD(T) - 1);
D(T) = (1-DELTA)** (ORD(T) - 1);
PV = 200*SUM(T, R(T)) + 90*(4*RTERM/(1+RHO))
;
```

```
$INCLUDE BASIC.GEN
SOLVE BASIC USING MCP;
```

```
PVUTIL = SUM(T, X.L(T)*R(T)) + (X.L('25')*R('25'))/RHO;
```

```
DISPLAY PVUTIL;
```

```
CONSUME(T) = X.L(T);  
INVEST(T) = I.L(T);  
KSTOCK(T) = K.L(T);  
SOLUTION(T, "X") = X.L(T);  
SOLUTION(T, "I") = I.L(T);  
SOLUTION(T, "K") = K.L(T);
```

```
$LIBINCLUDE XLDUMP SOLUTION SOL2.xls SHEET1!K2
```

INF.GMS

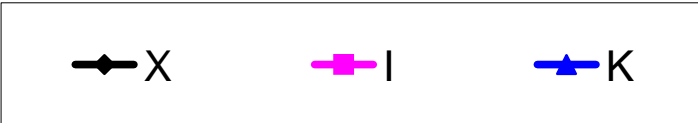
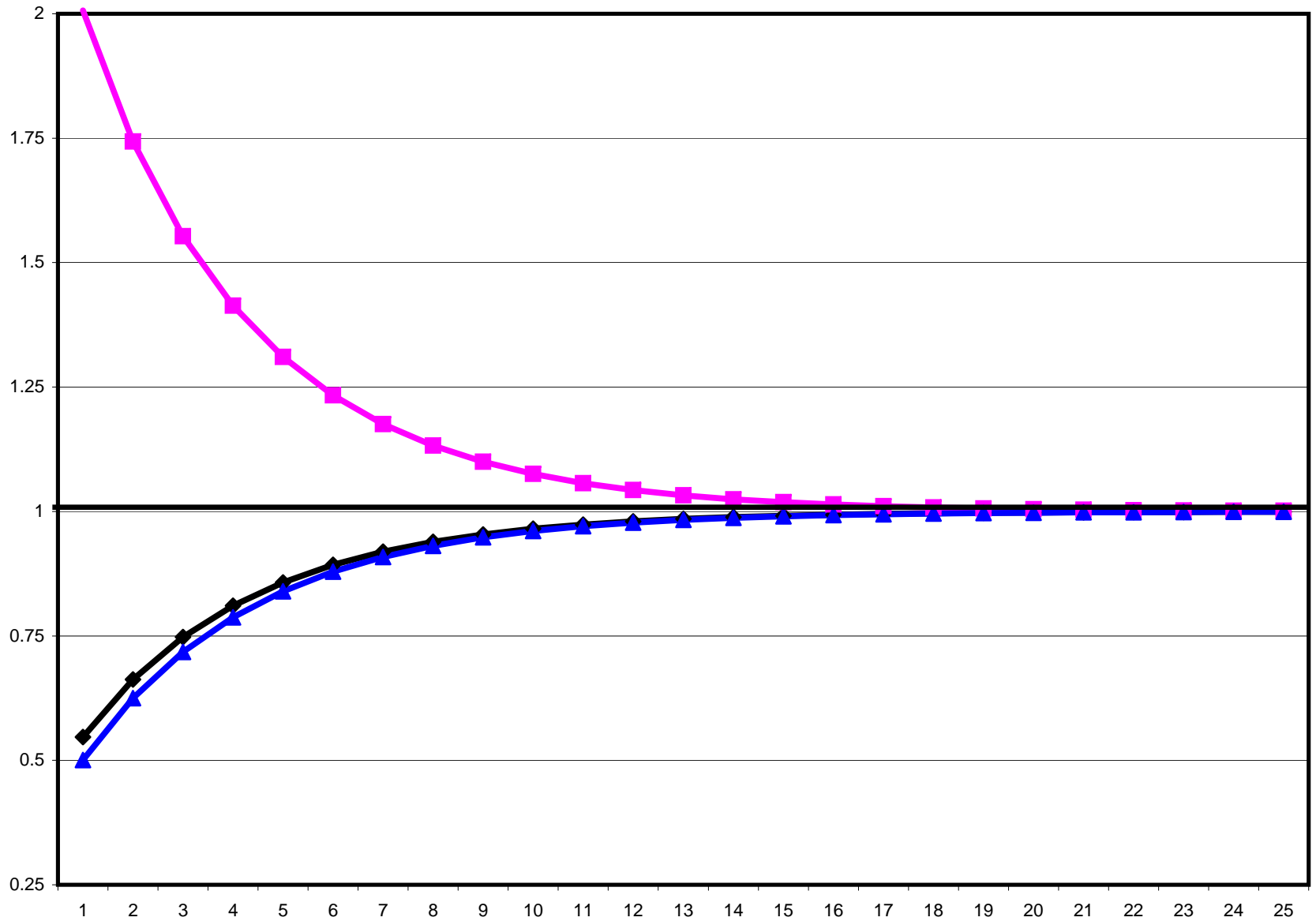
BENCHMARK  
(INITK = 90, RHO = 0.2)

INITK = 30

INITK = 90 RHO = 0.1

	X	I	K		X	I	K		X	I	K
1	1	1	1	1	0.54686	2.00673	0.50067	1	0.91656	1.49973	1.04997
2	1	1	1	2	0.66269	1.74314	0.62492	2	0.94617	1.44587	1.08956
3	1	1	1	3	0.74762	1.55314	0.71774	3	0.96957	1.40344	1.12095
4	1	1	1	4	0.81058	1.4137	0.78734	4	0.98807	1.36994	1.14585
5	1	1	1	5	0.85756	1.31037	0.83964	5	1.00273	1.34346	1.16561
6	1	1	1	6	0.89273	1.23332	0.87901	6	1.01435	1.32249	1.1813
7	1	1	1	7	0.91915	1.17566	0.90867	7	1.02357	1.30588	1.19376
8	1	1	1	8	0.93901	1.13237	0.93104	8	1.03089	1.29271	1.20365
9	1	1	1	9	0.95398	1.09983	0.94792	9	1.03669	1.28226	1.21151
10	1	1	1	10	0.96525	1.07533	0.96066	10	1.0413	1.27397	1.21776
11	1	1	1	11	0.97376	1.05686	0.97028	11	1.04496	1.26738	1.22272
12	1	1	1	12	0.98018	1.04294	0.97755	12	1.04787	1.26216	1.22666
13	1	1	1	13	0.98503	1.03243	0.98304	13	1.05018	1.25801	1.2298
14	1	1	1	14	0.98869	1.0245	0.98718	14	1.05202	1.25471	1.23229
15	1	1	1	15	0.99146	1.01851	0.99031	15	1.05348	1.25209	1.23427
16	1	1	1	16	0.99354	1.01398	0.99268	16	1.05463	1.25001	1.23584
17	1	1	1	17	0.99512	1.01057	0.99447	17	1.05555	1.24836	1.2371
18	1	1	1	18	0.99631	1.00799	0.99582	18	1.05629	1.24706	1.23809
19	1	1	1	19	0.99721	1.00605	0.99684	19	1.05687	1.24602	1.23888
20	1	1	1	20	0.99789	1.00458	0.99762	20	1.05733	1.2452	1.23952
21	1	1	1	21	0.99841	1.00348	0.9982	21	1.05769	1.24456	1.24002
22	1	1	1	22	0.99879	1.00266	0.99865	22	1.05798	1.24406	1.24043
23	1	1	1	23	0.99908	1.00205	0.99899	23	1.05821	1.24368	1.24075
24	1	1	1	24	0.9993	1.00163	0.99925	24	1.05839	1.2434	1.24102
25	1	1	1	25	0.99946	1.00134	0.99946	25	1.05853	1.24321	1.24124

Experiment: cut capital stock to 1/3 of steady-state value



Experiment: people are more patient, raise rho from 0.2 to 0.1

