Review of consumer theory: Chapter 3

Why? Trade depends on both production and consumption, so we need to represent demand.

Welfare gains and losses are about what happens to aggregate and individual consumption as a result of trade or trade policy.

We also want to be able to analyze distribution of welfare: who gains and who loses?

To do this we start with a single consumer optimization.

Standard conception
Consumer has a utility function that she maximizes with respect to an income or budget constraint.

Max U(X,Y) subject to $I = p_x X + p_y Y$

In the text this is represented as a Lagrangean problem with constrained maximization. It can be solved to get standard *Marshallian* demand functions.

But we can easily figure this out another way.

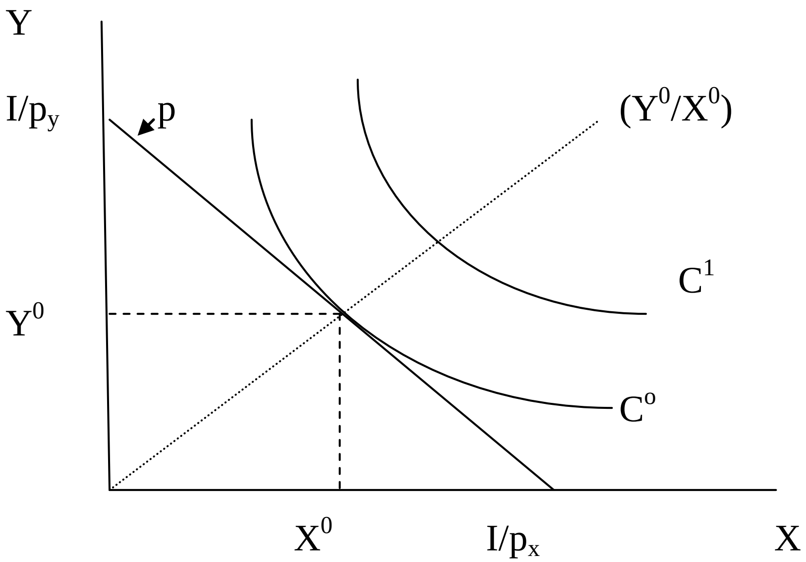
The budget constraint can be rewritten as $$Y = \frac{I}{p_y} - \frac{p_x}{p_y} X$$

This is the straight line in diagram below and in Figure 3.1.

Meanwhile we derive the individual's *indifference curve* as follows:

$$0 = dU = MU_x dX + MU_y dY \quad \text{from which} \quad -\frac{dY}{dX} = \frac{MU_x}{MU_y} = MRS$$

Here, MRS is the slope of the indifference curve. In the standard view these are convex in (X,Y), reflecting diminishing MRS. As less Y and more X is consumed for given utility, the marginal utility of X falls (of Y rises).



Note the (absolute) slope of the budget constraint is $p = p_x/p_y$. The equilibrium shown is the minimum cost of consuming X and Y to achieve utility level C^0. This is the *Hicksian* approach to demand functions: find minimum cost of a particular utility level:

$$\min p_x X + p_y Y \ subject \ to \ U(X,Y) = \bar{U}$$

In most situations these approaches are identical. The latter approach is better for thinking about welfare changes.

A key thing here is that individual demands depend on income and relative prices:

$$D_{xi} = D_{xi}(\frac{p_x}{p_y}, I_i)$$

Another point: the ray from the origin gives equilibrium consumption. Along this ray as income expands for fixed prices we find the *income-expansion path* (or Engel Curve). If the utility function is homogeneous (such as Cobb-Douglas) then this path is a straight line.

Now suppose there are 2 or more consumers, which we want to permit in order to study income distribution. How can we aggregate their preferences?

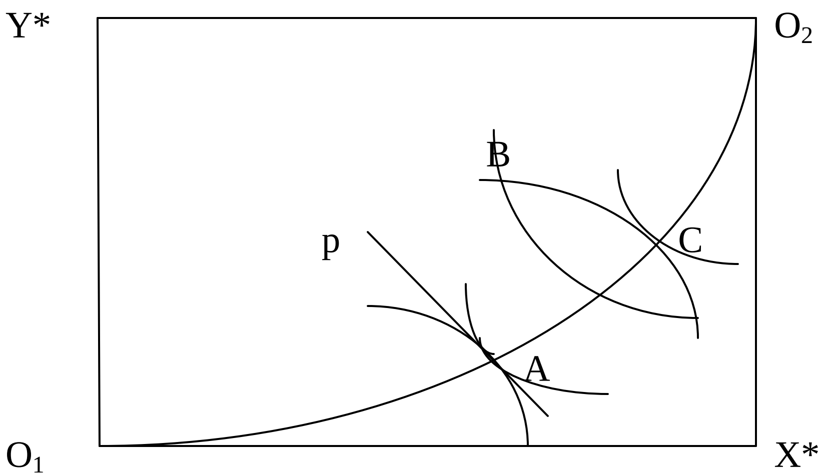
First, we see from the diagram that MRS = p.

Since this situation must be true in equilibrium for all consumers, we have the condition for Pareto Efficiency in consumption for n consumers:

$$p = MRS_1 = \ldots = MRS_n$$

If this were not true one could reallocate consumption among consumers at the given prices to make at least one consumer better off without harming any other consumer.

One depiction is with a consumption Edgeworth Box along a contract curve between individuals 1 and 2:



Here, X* and Y* are the total amounts of goods available. The Box indicates how they are distributed between 1 and 2, with 2's origin at the upper right side. A is on the contract curve for price ratio p; it is Pareto Efficient. B is not efficient since we can raise 1's utility and not reduce 2's utility by moving to C. Note if these are homogeneous indifference curves the price line at C must be steeper than at A.

Income distribution (or welfare distribution) changes along the contract curve. As you move up to the right, 1 gets more consumption and 2 gets less consumption.

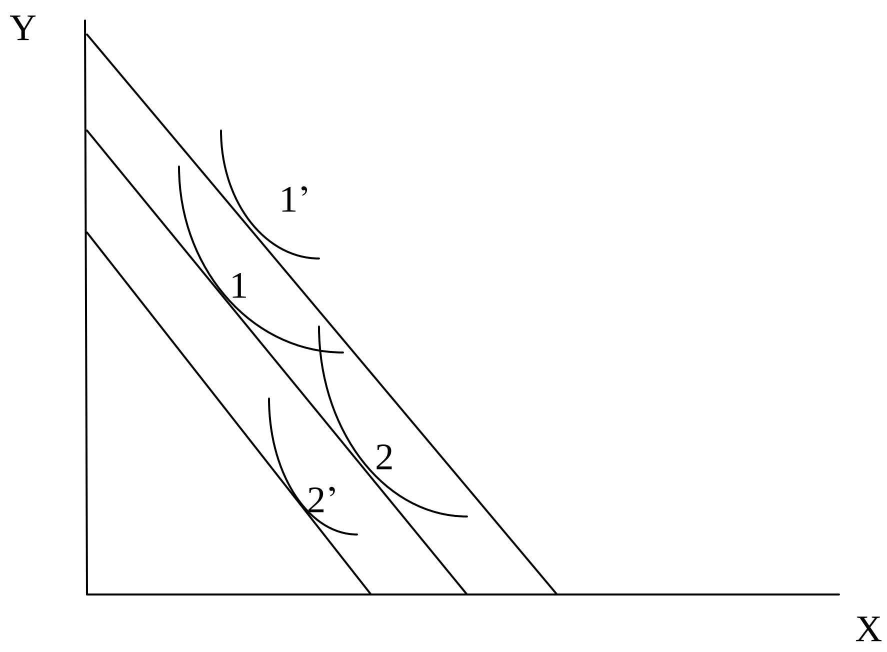
This raises a fundamental question about aggregating preferences to get a *national* or *community demand curve*. We want to do that in order to analyze welfare changes from trade policy or taxes, etc.

That is, we need an aggregate demand curve for X and Y:

$$D_x = D_x(p, NI) \ \ where \ \ NI = \sum_1^n I_i$$

with the usual properties: demand for X rises as p falls and it rises with NI (if normal good) or falls as NI goes up (if inferior good).

This is possible only if income distribution does not matter in determining total consumption of X and Y. In general this is not true. Consider this case (and Figure 3.5):



Start with the same incomes for 1 and 2. Now take income away from 2 and give it to 1. If you draw in the lines indicating each person's consumption change you should see that they do not balance. So even though NI has not changed, total demands for X and Y have changed.

In this case tastes are not the same between 1 and 2 and in general that would mean income distribution matters for demand. But Figure 3.4 in text shows a case where they have identical demands but income still matters because their utility functions are not homogeneous. There as a consumer gets richer she consumes relatively less Y and more X. So shifting income between them will change aggregate demands.

A simple numerical example.

Suppose 2 goods, Scones (S) and CDs (C) and 3 people (picked in class). Let there be 30 scones and 9 CDs with different initial distributions. Take away one CD per person. How many scones for the same utility?

	Case A				Case B			
Person	S_0	C_0	S_1	C_1	S_0	C_0	S_1	C_1
1	10	2		1	15	3		2
2	7	2		1	10	3		2
3	13	5		4	5	3		2
Total	30	9		6	30	9		6

Here we made person 3 poorer and 1 and 2 richer in Case B. For income distribution not to matter the total S demand can't change from A to B.

Under what circumstances does "aggregate demand" exist in the sense that income distribution does not matter? The following cases:

1. “Robinson Crusoe” economy with a single consumer.
2. “Representative agent” model where all people have the same preferences (utility function) and income (so distribution never changes). This is the “macroeconomic model” widely in use.
3. All people have same preferences and their indifference curves are either
    a. Homogeneous (income-expansion paths are straight rays from origin).
    b. Quasi-homogeneous (income-expansion paths are straight rays but displaced from the origin, as in Figure 3.6.

    In either a. or b. equal income redistribution from 1 to 2 does not affect demands for goods X and Y. (It would be worthwhile to draw some diagrams to be sure about that.)

In these cases we say aggregate demand functions exist and we can use them to analyze trade. This is the so-called *positive* interpretation of the demand side of the aggregate economy.

But we generally want to say more about the welfare impacts of trade and get a *normative* interpretation. So we can ask: is there an aggregate utility function U = U(X,Y) with the standard properties:

1. Convex *community indifference curves* that are convex, non-intersecting;
2. A movement from lower to higher CIC implies higher social welfare.

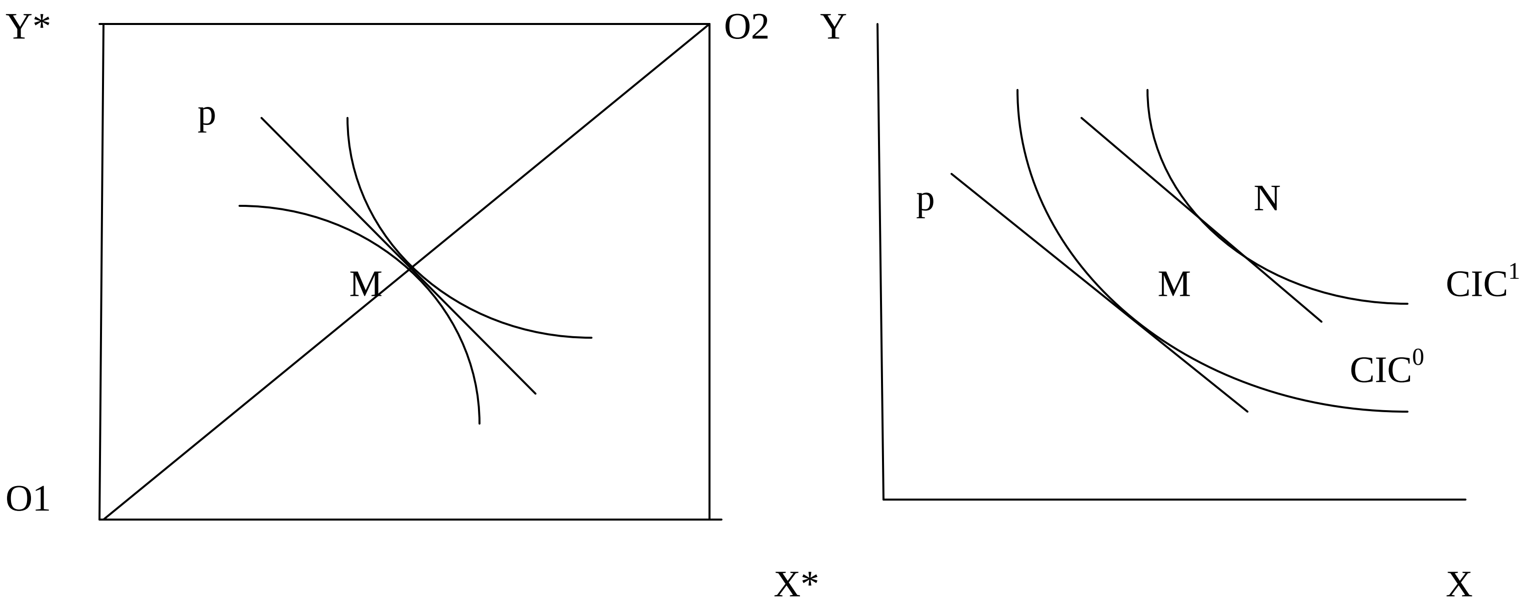
How would we define a CIC in general? It shows combinations of X and Y along which society feels equally well off. In general this only makes sense if there is no redistribution of welfare among individuals as we move along a CIC. So we might define: A CIC shows combinations of X and Y along which each individual feels equally well off, so that society as a whole has identical welfare.

When does this happen?

1. Robinson Crusoe economy (not useful for trade analysis).
2. RA model with identical tastes and incomes.

How does case 3 give us a national utility function?

With the same incomes, the Edgeworth Box is split in half between 1 and 2 so there really is only one initial distribution. And with identical tastes, that determines the MRS for both individuals 1 and 2.



In this case both have half the available goods and p = MRS for both 1 and 2. In fact, the slope of either individual indifference curve then determines the slope of the CIC in 2d diagram.

How do we know that moving from lower to higher CIC implies higher welfare? Because both 1 and 2 share equally in the higher income at N than at M.

So in this case we do have a well-defined community utility function U(X,Y) with standard CICs.

Thus, the RA model works both in positive and normative cases.

But for virtually any other situations, the distribution of income will change as national incomes change. For example, if preferences for 1 and 2 are identical and homogeneous but income distribution can change, we can draw CICs as above but in moving from M to N we might get 2 gains and 1 loses.

In that case what can we say? We mention the idea of *compensation*. If a policy change raises national income so that the winner(s) can compensate the loser(s) so that no one is worse off and at least one person is better off, society has gained. This will always be true if preferences are identical and homogeneous (or quasi-homogeneous) and we are willing to draw CICs and use them for welfare analysis.

But it can hold in more general circumstances also.

Example of a compensation policy in the United States: Trade Adjustment Assistance.