Chapter 8

DIFFERENCES IN FACTOR ENDOWMENTS I: THE HECKSCHER-OHLIN MODEL

8.1 The Heckscher-Ohlin model: an intuitive approach

We now turn to a second determinant of trade, differences in factor endowments across countries, combined with differences in factor intensities across goods. In terms of the no-trade model of Chapter 6, we are now going to re-establish the assumption that countries have identical technologies, but relax the assumption that they have identical relative factor endowments. Obviously, this will require that we add a second factor of production. All of the other assumptions of the no-trade model remain in place.

The development of this theory is generally associated with two Swedish economists, Heckscher and Ohlin. Sometime the term Heckscher-Ohlin theory is used to refer to all models in which differences in factor endowments are the driving force of trade. Sometimes it is used in a much more narrow sense to refer specifically to a model with two goods and two factors and in which both factors are mobile between or useful in both industries. We will focus on the two-good, two-factor case here and comment on more general models at the end of the next chapter.

The two-factor Heckscher-Ohlin model is considerably richer than the Ricardian model and allows for more realistic predictions. First, because the production frontier is "bowed out" as discussed in Chapter 2, countries will have much less of a tendency to specialize. In the two-country Ricardian model of the previous chapter, at least one country must be specialized. Second, Heckscher-Ohlin allows for interesting and important income distributional effects from trade. In particular, the owners of the factors will be in conflict in their views about the desirability of liberal trade versus trade protection. This is an important first step in understanding the political economy of trade protection.

Two goods are produced from two factors which are in fixed (or "inelastic") supply. We adopt the convention of Chapter two that when a variable refers to both a good and factor, the first subscript is the industry and the second denotes the factor.

$$X_1 = F_1(V_{11}, V_{12}) \qquad X_2 = F_2(V_{21}, V_{22}) \tag{8.1}$$

$$\bar{V}_1 = V_{11} + V_{21} \qquad \bar{V}_2 = V_{12} + V_{22}$$

Assume that industry X_1 is intensive in the use of factor V_1 when competitive firms choose optimally, and assume that country h is relatively abundant in factor V_1. We are assuming that

$$\frac{V_{11}}{V_{12}} > \frac{V_{21}}{V_{22}} \quad \text{in both countries and} \quad \frac{\bar{V}_{h1}}{\bar{V}_{h2}} > \frac{\bar{V}_{f1}}{\bar{V}_{f2}} \tag{8.2}$$

Figures 8.1 and 8.2 present a very special case in order to provide the intuition behind the main result of the Heckscher-Ohlin model. Points H and F denote the total factor endowments of countries h and f respectively. Isoquants through these endowment points denote the amounts of X_1 and X_2 that would be produced if the country only produced that one good. Thus $\bar{X}_{h2}$ and $\bar{X}_{h1}$ are the endpoints of country h's production frontier, shown in Figure 8.2. Similarly for country f. In this special case, country h can produce absolutely more X_1 and country f can produce absolutely more X_2 than country h. Comparative advantage is derived from the intersection between relative factor endowments between countries and relative factor intensities between industries. Each country has a comparative advantage in the good using intensively its abundant factor.

Figure 8.1 Figure 8.2

In drawing Figure 8.2, we also make symmetry assumption that the countries are mirror imagines of one another and that they spend half their income on each good. In this special case, the autarky equilibria for the two countries are at points A_h and A_f in Figure 8.2 respectively. In autarky equilibrium, each country has the same utility, but each country consumes more of the good in which it has a comparative advantage, the good using intensively its abundant factor. Note that each country will also have a low relative price for the good using intensively its abundant factor. Now permit trade. Each country will export the good using intensively its abundant factor, producing at points T_h and T_f for countries h and f respectively, and both consuming at the same point D in figure 8.2. This is the Heckscher-Ohlin theorem.

> Heckscher-Ohlin theorem: each country exports the good using intensively its relatively abundant factor.

8.2 The Heckscher-Ohlin theorem: a more formal approach

Figures 8.1 and 8.2 show a very special case. The result is much more general than this however and that is what we shall show in this section. Consider a single country and assume for the moment that both industries are producing at world prices p_1 and p_2. One of the requirements for general equilibrium that we discussed in Chapter 4 is that profits are zero. Zero profit conditions give

$$\begin{bmatrix} c_1(w_1, w_2) \\ c_2(w_1, w_2) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \tag{8.3}$$

where again, a_{ij} is the *optimal* amount of factor j used in industry i to produce one unit of good X_i. Using the envelope theorem and its application in Shephard's lemma from Chapter 2 and equation (2.29), we can differentiate the first equation of (8.3) and arrive at the following result.

$$dc_1 = a_{11}dw_1 + a_{12}dw_2 + \left[w_1 da_{11} + w_2 da_{12}\right] = a_{11}dw_1 + a_{12}dw_2 = dp_1 \tag{8.4}$$

The term in brackets is zero: with the a_{ij}'s chosen optimally, small changes in these values have no effect on cost; this is Shepard's lemma. There is a similar equation for industry 2, and (8.4) implies that the right-hand equation of (8.3) also holds in differentials.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dw_1 \\ dw_2 \end{bmatrix} = \begin{bmatrix} dp_1 \\ dp_2 \end{bmatrix} \tag{8.5}$$

Invert this two-equation system. By a formula typically known in economics as Cramer's Rule, the inverse mapping is given by

$$\begin{bmatrix} a_{22}/D & -a_{12}/D \\ -a_{21}/D & a_{11}/D \end{bmatrix} \begin{bmatrix} dp_1 \\ dp_2 \end{bmatrix} = \begin{bmatrix} dw_1 \\ dw_2 \end{bmatrix} \tag{8.6}$$

where D is the determinant of the a_{ij} matrix in (8.5). As in Figures 8.1 and 8.2, assume that X_1 is V_1

intensive, so that

$$\frac{a_{11}}{a_{12}} > \frac{a_{21}}{a_{22}} \quad \Rightarrow \quad a_{11}a_{22} > a_{21}a_{12} \qquad a_{11}a_{22} - a_{21}a_{12} \equiv D > 0 \tag{8.7}$$

The determinant D in (8.6) is positive, so the diagonal elements are positive and the off-diagonal elements are negative.

For the rest of the section, let $p_2 = 1$ be numeraire and so p_1 is the relative price of good 1 in terms of good 2. (8.6) then implies the following relationship between factor prices and goods prices.

$$\left[\frac{dw_1}{dp_1}\right]_{dp_2 = 0} > 0 \qquad \left[\frac{dw_2}{dp_1}\right]_{dp_2 = 0} < 0 \tag{8.8}$$

Intuitively, a rise in the price of good X_1 will lead to an increase in the price of the factor used intensively in good X_1, which is factor V_1 by assumption. This in turn is associated with an optimal movement around an isoquant in each industry, substituting away from the factor whose price has increase and toward the factor whose price has decreased. Thus (8.8) in turn implies

$$\frac{da_{11}}{dp_1} < 0 \qquad \frac{da_{12}}{dp_1} > 0 \qquad \frac{da_{21}}{dp_1} < 0 \qquad \frac{da_{22}}{dp_1} > 0 \qquad (dp_2 = 0) \tag{8.9}$$

We will discuss this in more detail later in the section on the Stolper-Samuelson theorem.

A second requirement for general equilibrium discussed in Chapter 4 is factor-market clearing. Let V_1 and V_2 denote the *exogenous* supplies of factors 1 and 2 (we drop the overbar), and X_1 and X_2 are the *endogenous* production levels. Factor market clearing requires:

$$\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{8.10}$$

Note that the matrix here is just the transpose $[a_{ij}]'$ of the matrix $[a_{ij}]$ in (8.3) and (8.5). Invert this mapping.

$$\begin{bmatrix} a_{22}/D & -a_{21}/D \\ -a_{12}/D & a_{11}/D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{8.11}$$

Now divide the first equation by the second, and divide the numerator and the denominator by V_1.

$$\frac{X_1}{X_2} = \frac{a_{22} - a_{21}\dfrac{V_2}{V_1}}{-a_{12} + a_{11}\dfrac{V_2}{V_1}} \tag{8.12}$$

Keeping in mind (8.9), equation (8.12) establishes several crucial things.

(a) the production ratio X_1/X_2 rises with p_1/p_2. This follows from (8.9): the relative V_2 intensity of both industries rise with an increase in the relative price of p_1 (a_{22} and a_{12} rise and a_{21} and a_{11} fall) . The numerator of (8.12) gets larger and the denominator gets smaller. This should be intuitive: the relative supply of good X_1 rises with the relative price of X_1.

(b) the price ratio at which a country just begins to produce X_1 (the numerator of (8.12) equals zero), is higher in the V_2 abundant country. The higher V_2/V_1, the higher a_{22}/a_{21} must be for the numerator to be zero, which in turn requires a higher relative price of p_1 by (8.9).

(c) the price ratio at which a country stops producing X_2 (the denominator of (8.12) equals zero) is higher in the V_2 abundant country. The higher V_2/V_1, the higher a_{12}/a_{11} must be for the denominator to be zero, which in turn requires a higher relative price of p_1.

These results imply the general-equilibrium supply curves in Figure 8.3, which graphs p_1/p_2 against X_1/X_2. The upper curve is for a relatively V_2 abundant country f and the lower curve is for a relatively V_1 abundant country h.

Figure 8.3

Now add demand, and assume that both countries have identical and homogeneous preferences. Then both countries have the identical demand curve shown by D in Figure 8.3: the ratio in which goods are consumed depends only on relative prices and not on size or income. The autarky equilibria for the two countries are at points F for country f and H for country h in Figure 8.3. Autarky price ratios are given by p_f^a and p_h^a in Figure 8.3.

The final piece of the puzzle is shown in Figure 8.4. With each country having relative low price for the good using intensively the country's relatively abundant factor as in Figure 8.3, the excess demand curves are shown in Figure 8.4 as F and H for countries f and h respectively. Free trade equilibrium is at price p^*. With trade, we then have the Heckscher-Ohlin theorem: each country exports the good using intensively the country's abundant factor.

Figure 8.4

Figures 8.3 and 8.4 are the more general versions of Figures 8.1 and 8.2, or alternatively Figures 8.1and 8.2 are special cases of 8.3 and 8.4.

8.3 The factor-price-equalization theorem

There are three additional results associated with the Heckscher-Ohlin model. One, the factor-price-equalization theorem, relates to the two-country trade model, while the third and fourth strictly speaking are just comparative-statics results for a single country, but they have important trade implications. All three depend on the restrictive assumption that a country produces both goods and the factor-price-equalization theorem requires that both countries produce both goods (countries are diversified, or non-specialized). We will begin with the factor-price-equalization theorem, in part because it allows us to assess the plausibility of the non-specialization assumption.

Factor-price equalization (henceforth FPE) is the result that, under very restrictive assumptions, trade will equalize the price of each factor of production across countries even though the factors themselves are not traded: goods trade alone may equalize factor prices. Intuitively, trade in goods indirectly creates competition for factors embodied in the goods trade. The basic idea behind factor-price equalization (henceforth fpe) is seen in equation (8.3). (1) If countries have identical constant-returns technologies, then their cost functions for goods are the same, and the same two equations in (8.3) apply to both countries. So the mapping between two goods prices and two factor prices is identical across countries. Then assume that (2) trade is completely costless so that goods prices are equalized between countries, and (3) both countries produce both goods in free-trade equilibrium. Under these three assumptions, it will be true that the price of each factor is equalized across countries.[1]

Figure 8.5 presents this graphically. Instead of isoquants, it uses unit-value isoquants; that is, combinations of V_1 and V_2 that yield one unit value of output, say \$1 of output. The position of unit-value isoquants thus depends on prices: the higher a good's price, the closer is the unit-value isoquant to the origin, since less physical output is needed to generate \$1 of output. Figure 8.5 shows unit-value isoquants for X_1 and X_2, where the prices that position these isoquants are exogenous for the moment. If assumptions (1) and (2) of the previous paragraph hold, then these two unit-value isoquants will be identical in the two countries. This is step one in the argument. Step two in the proof is that there is one unit-value isocost line, combinations of V_1 and V_2 that cost \$1, tangent to the two unit-value isoquants. This is shown in Figure 8.5 as the straight iso-cost line . At each point on this line, total factor cost is \$1. If both countries are producing both goods, assumption (3) above, then the price of each factor will be identical in both countries.

Figure 8.5

Step three in the argument is that the tangencies between the unit-value isoquants and unit-isocost line establish the optimal V_2/V_1 ratios in the two industries. These are denoted a_{22}/a_{21} and a_{12}/a_{11} in Figure 8.5. The final step is to note that if the countries V_2/V_1 endowment ratio lies inside the cone spanned by the two optimal factor ratios, then it is possible to divide up the endowment so that each industry produces with its optimal V_2/V_1 ratio. The cone is often referred to as the cone of diversification.

Consider point E in Figure 8.5. Point E is the sum of the vectors $0V^2$ and $0V^1$, so the latter two give the factors allocated to industries X_2 and X_1 respectively when the optimal factor ratios are used. When the endowment point lies inside of the cone of diversification, the country will produce both goods. If this is true for both countries so that (1), (2), and (3) are all satisfied, then we have factor-price equalization.

> Factor-price equalization theorem: If (1) two undistorted competitive economies have identical technologies, (2) costless trade equalizes commodity prices between countries and (3) both countries produce both goods in equilibrium, then the price of each factor is equalized across countries.

If the country's endowment is at a point like E^2 in Figure 8.5, it will produce only X_2 with factor prices given by the slope of the X_2 isoquant at that point. If its endowment is at E^1, it will produce only X_1 with factor prices given by the slope of the X_1 isoquant at that point. This establishes the intuition why non-specialization is needed for the theorem.

The limitation of this analysis is that non-specialization is an endogenous outcome of trade, it really should not be imposed as an assumption. In order to treat it appropriately as endogenous, we turn to a world Edgeworth box in Figure 8.6, using a technique made popular by Dixit and Norman (1980). On the vertical axis is the total world endowment of V_2 and the total world endowment of V_1 is on the horizontal axis. The origin for country h is in the lower left or southwest corner and the origin for country f is at the northeast corner.

Figure 8.6

Consider the experiment in which there is only a single country and a single market for each factor, and then solve for the competitive equilibrium, which is referred to as the integrated world equilibrium. Then observed the optimal V_2/V_1 ratios chosen in the two industries and graph these vectors from each origin in Figure 8.6. This will produce the parallelogram shown in Figure 8.6. Now re-introduce the two separate countries, assuming again (1) and (2) above. If the endowment point E which divides the total world endowment between the two countries lies inside the parallelogram, then the free-trade equilibrium supports factor prices equalization. In term of Figure 8.5, each country's endowment lies inside the cone of diversification.

8.4 The Rybczynski theorem

As note above, there are two additional results which, strictly speaking are comparative-statics results for an individual country, but they have important trade and policy implications. The Rybczynski theorem concerns the effects of changing endowments on output holding commodity prices constant. It is valid only under the assumption that a country is non-specialized, producing both goods.

The Rybczynski theorem begins with an insight from the FPE theorem and Figures 8.5 and 8.6. If commodity prices are held constant and the country is producing both goods, then factor prices are determined and constant. This in turn determines the optimal V_2 / V_1 ratios to use in the two industries as in Figure 8.5. The question addressed by the theorem can be thought of as the experiment of moving the endowment point around in Figure 8.5.

The theorem is illustrated in Figure 8.7, which is an Edgeworth box for a single country. $\bar{V}_2$ and $\bar{V}_1$ are its initial endowments, and A^0 represents its initial equilibrium at fixed commodity prices. Now increase the country's endowment of V_1 by an amount ΔV_1. The new origin for good X_2 is at O_2'. With optimal V_2 / V_1 ratios pinned down by the fixed prices, the new allocation in the expanded Edgeworth box must be at point A^1, which preserves the V_2 / V_1 ratios in the two industries. Note especially that the amount of factors allocated to the X_2 industry shrinks from O_2A^0 initially to $O_2'A^1$ after more V_1 is added. Factor allocation to X_1, the industry intensive in V_1, expands more than in proportion to the increase V_1 while the allocation to X_2 shrinks. This in turn leads to the Rybczynski theorem.

Figure 8.7

> Rybczynski Theorem: Holding commodity prices constant, an increase in the endowment of one factor leads to a more than proportional increase in the output of the good using that factor intensively and a fall in the output of the other good.

The result is illustrated in Figure 8.8, where A^0 corresponds to A^0 in Figure 8.7 and similarly for A^1 in the two Figures. The biased change in the factor endowments leads to an even greater biased change in outputs.

Figure 8.8

A more formal treatment is as follows. Consider (8.10) above. Since the a_{ij} depend only on commodity prices (which fix factor prices), this also holds in differential form. The total derivative of the first equation in (8.10) is given by

$$a_{11} dX_1 + a_{21} dX_2 = dV_1 = \left[\frac{V_{11}}{X_1}\right] dX_1 + \left[\frac{V_{21}}{X_2}\right] dX_2 \tag{8.13}$$

Dividing through by the total factor endowments V_1 and V_2 (drop the overbar notation), the two equations for the two factors can be written as

$$\left[\frac{V_{11}}{V_1}\right]\frac{dX_1}{X_1} + \left[\frac{V_{21}}{V_1}\right]\frac{dX_2}{X_2} = \frac{dV_1}{V_1} \tag{8.14}$$

$$\left[\frac{V_{12}}{V_2}\right]\frac{dX_1}{X_1} + \left[\frac{V_{22}}{V_2}\right]\frac{dX_2}{X_2} = \frac{dV_2}{V_2} \tag{8.15}$$

The terms in square brackets in (8.14) and (8.15) are shares of each factor used in each industry and hence they are all between zero and one. Using notation and methodology developed in a classic article by Jones (1967), denote the share of factor j which is used in good i with the parameter λ_{ij} and let proportional changes in a variable by denoted by a hat over the variable. Then our two equations become:

$$\begin{bmatrix}\lambda_{11} & \lambda_{21}\\ \lambda_{12} & \lambda_{22}\end{bmatrix}\begin{bmatrix}\hat{X}_1\\ \hat{X}_2\end{bmatrix} = \begin{bmatrix}\hat{V}_1\\ \hat{V}_2\end{bmatrix} \tag{8.16}$$

Now invert the two equation system.

$$\begin{bmatrix}\lambda_{22}/D & -\lambda_{21}/D\\ -\lambda_{12}/D & \lambda_{11}/D\end{bmatrix}\begin{bmatrix}\hat{V}_1\\ \hat{V}_2\end{bmatrix} = \begin{bmatrix}\hat{X}_1\\ \hat{X}_2\end{bmatrix} \tag{8.17}$$

$$D_\lambda = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21} > 0 \quad where \quad \frac{\lambda_{22}}{\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}} > 1, \quad 1 > \lambda_{ij} > 0$$

Examining the determinant D and remembering that each λ_{ij} is between zero and one, the magnitudes and signs of the mapping in (8.17) are as follows.

$$\lambda^{-1}\begin{bmatrix}\hat{V}_1\\ \hat{V}_2\end{bmatrix} = \begin{bmatrix}>1 & <0\\ <0 & >1\end{bmatrix}\begin{bmatrix}\hat{V}_1\\ \hat{V}_2\end{bmatrix} = \begin{bmatrix}\hat{X}_1\\ \hat{X}_2\end{bmatrix} \tag{8.18}$$

The Rybczynski theorem gives the effect of increasing each factor individually.

$$\hat{X}_1 > \hat{V}_1 > 0 > \hat{X}_2 \qquad \hat{X}_2 > \hat{V}_2 > 0 > \hat{X}_1 \tag{8.19}$$

Jones referred to this as a “magnification” effect. A change in the endowment of one factor holding prices constant leads to magnified changes in outputs.

While the Rybczynski theorem has nothing directly to do with trade, it does have important implications for a number of empirical and policy issues connected with trade. First of all, it emphasizes that, at least for a fairly small country that has little influence over world prices, large changes in the country’s factor endowment may be absorbed by changing the composition of output with very little effect on factor prices. This is very different from the way that labor economists think about the economy using the partial-equilibrium tools of supply and demand. The general-equilibrium Rybczynski theorem says that at constant world prices, the labor (or capital) demand curve is flat and big changes in supply may have little effect on the wage (or return to capital).

Second, the Rybczynski theorem has been used to help understand large changes in the composition of output over the last two decades, particularly in East Asia. Some writers have talked about an "Asian miracle" referring to large shifts from traditional agriculture to manufacturing. But trade economists know that, with sharply falling birth rates and very high savings rates, East Asia changed its factor endowment in a very biased way away from unskilled labor and toward capital and skilled labor and the very biased change in the composition of output is perfectly consistent with the Rybczynski theorem. There is no need to appeal to miracles.

8.5 The Stolper-Samuelson theorem

A final theorem connected with the Heckscher-Ohlin model is the Stolper-Samuelson theorem. Technical is "dual" to the Rybczynski theorem: the latter is a relationship in quantities, the former a relationship in prices. If one is true, the other must be true. As in the case of the Rybczynski theorem, the Stolper-Samuelson theorem is a magnification effect.

> Stolper-Samuelson Theorem: Holding factor endowments constant, an increase in the price of one good leads to a more than proportional increase in the price of the factor used intensively in producing that good and to a fall in the price of the other factor.

A graphical presentation of the result is given in Figures 8.9 and 8.10. Figure 8.9 is an Edgeworth box for a single economy as in Figure 8.6 above. Let A^0 denote the initial equilibrium in the factor markets. Now assume that the price of good X_1 increases holding p_2 constant. There will now be positive profits to be earned in the X_1 industry at initial factor prices and holding those prices constant for the moment, the X_1 producers would want to expand along ray a_{12}/a_{11} holding the factor use ratio constant at its optimal value. X_2 would contract along ray a_{22}/a_{21}. But this cannot be an equilibrium. There would be excess demand for factor V_1 (used intensively in X_1) and excess supply of factor V_2. The price of the former must rise to clear the factor market and the price of V_2 must fall. The new equilibrium is at A^1 in Figure 8.9 with factor markets once again clearing.

Figure 8.9

The change in factor prices may be more clear from Figure 8.10, which shows the movement around a single isoquant. A^0 in Figure 8.10 corresponds to A^0 in Figure 8.9 and similarly for point A^1 in the two figures. The increased demand for V_1 and the fall in demand for V_2 at initial factor prices is re-equilibrated by a rise in w_1/w_2 in both figures and a rise in the V_2/V_1 ratios in both industries.

Figure 8.10

This graphical treatment only makes clear that the relative price w_1/w_2 rises when p_1 rises. But the theorem is stronger than that, it says that w_1 must rise more than in proportion to p_1; that is, the real price of w_1 rise and factor V_1 owners are better off no matter what they choose to consume. Refer back to the two-equation system in (8.5). We can transform this mapping into proportion changes that we did with the Rybczynski theorem. For the first equation, we have

$$\left[\frac{V_{11}}{X_1}\right]dw_1 + \left[\frac{V_{12}}{X_1}\right]dw_2 = dp_1 \quad => \quad \left[\frac{w_1 V_{11}}{p_1 X_1}\right]\frac{dw_1}{w_1} + \left[\frac{w_2 V_{12}}{p_1 X_1}\right]\frac{dw_2}{w_2} = \frac{dp_1}{p_1} \qquad (8.20)$$

The terms in brackets are the shares of each factor's earnings in the total revenue of the industry and all are between zero and 1. Using θ_{ij} for the share of factor j's income in industry i's revenue, these can be written as

$$\begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \end{bmatrix} = \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \end{bmatrix} \quad \textit{where} \quad D = \left[\theta_{11}\theta_{22} - \theta_{12}\theta_{21}\right] > 0, \quad 1 > \theta_{ij} > 0 \qquad (8.21)$$

The sign of the determinant of the share's matrix follows from the assumption that industry 1 is intensive in factor 1. Inverting the equations in (8.21), we have

$$\begin{bmatrix} \theta_{22}/D & -\theta_{12}/D \\ -\theta_{21}/D & \theta_{11}/D \end{bmatrix} \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \end{bmatrix} = \begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \end{bmatrix} \qquad (8.22)$$

which has the sign and magnitude pattern given by

$$\theta^{-1} \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \end{bmatrix} = \begin{bmatrix} >1 & <0 \\ <0 & >1 \end{bmatrix} \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \end{bmatrix} = \begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \end{bmatrix} \qquad (8.23)$$

As noted above, the Stolper-Samuelson theorem is given formally by the magnification relationships

$$\hat{w}_1 > \hat{p}_1 > 0 > \hat{w}_2 \qquad \hat{w}_2 > \hat{p}_2 > 0 > \hat{w}_1 \qquad (8.24)$$

A biased change in commodity prices has an even more bias change in factor prices, strongly redistributing income between groups of factor owners. With a rise in p_1, V_1 owners are better off even if they only want to consume X_1 and V_2 owners are worse off even if they only want to consume X_2.

Again, the Stolper-Samuelson theorem has nothing directly to do with international trade, but it does shed light on many external changes on an economy and the effects of domestic trade liberalization or protectionist policies on income distribution. These policies and external shocks change traded commodity prices and hence redistribute income inside the country.

The result is fundamental and often taken as a starting point for analyzing the political economy of trade policy to explain why, for example, some groups will vigorously lobby for protection and against liberalization even though this reduces the aggregate income of the country. A group who might be made worse off by liberalization may understandably have little interest in the argument that it increases national welfare. The latter result does imply that it must be possible to redistribute income following liberalization in a way that makes everyone better off. Unfortunately, how to do this is difficult to calculate in practice and difficult to implement for governments, so the ability of the gainers to compensate the losers is generally not very relevant to practical politics. Much more will be said about this throughout the book.

8.5 A caveat: factor-intensive reversal (may be skipped without loss of continuity)

Unfortunately, there is one complication that can invalidate much of what we said above. The problem is that the zero-profit conditions in (8.3) may not have a unique solution. Suppose that the unit-value isoquant discussed in Figure 8.3 look like those in figure 8.11. There are two crossings, and hence there are two cones of diversification with different factor prices. This can occur when the elasticities of substitution (curvature of the isoquants) are very different in the two industries: in the case shown, there is a zero elasticity of substitution in industry X1(production function (ii) in equation (2.3) of Chapter 2). If country f has its endowment in the upper cone at E_f and country h is at E_h, then both countries will produce both goods but they will have different factor prices and use different factor intensities even if commodity prices are equalized between countries. This is referred to as factor-intensity reverse: note that for country f, good X_2 is intensive in factor V_2, but for country h, X_1 is intensive in V_2. We will not discuss this further here, but simply note that an additional assumption is needed in order to rule this out for the above theorems to be valid. Ask your professor is you have further questions.

Figure 8.11

8.6 Summary: what you should know

This chapter has considered a model in which the only important difference between countries is in relative factor endowments. We assume that economies are alike in all other respects, including having equal access to constant-returns-to-scale technologies, sharing homogeneous preferences, and exhibiting no market or policy distortions. Thus, the model differs importantly from the technology-based theory of trade in the prior chapter. With this approach we established several important propositions about economic structure and trade patterns.

A country will export the commodity that uses the abundant factor more intensively, which is the prediction of the Heckscher-Ohlin theorem. Thus, comparative advantage is determined by the structure of factor endowments (or autarky factor prices) in conjunction with relative factor intensities of commodities.

A powerful insight about the effects of trade comes from the factor-price-equalization theorem, in which free trade in goods actually equalizes the relative and absolute prices of homogeneous factors internationally. The essential reason for this result is that trade in goods can substitute for trade in factors. The assumptions under which this theorem holds are severely restrictive and there are numerous reasons why we do not observe such equalization in practice. But there are important tendencies in that direction to the extent that international trade is the result of variations in factor endowments.

The Stolper-Samuelson theorem, which relates changes in commodity prices to changes in real factor prices, provides a fundamental prediction about the effects of trade (or impediments to trade) on the distribution of real incomes between (for example) capital and labor. Because free trade causes exports and imports to rise, it follows that the relatively abundant factor gains real income in each country and the scarce factor loses real income. The gains-from-trade theorem is relevant here, in that the economy overall enjoys a welfare rise in moving from autarky to free trade. However, the theorem will predict political conflicts within a country over trade liberalization versus protection in the absence of an explicit mechanism to redistribute gains.

The Rybczynski theorem, which relates changes in factor endowments to changes in commodity outputs, assuming constant commodity and factor prices, provides the theoretical basis for the Heckscher-Ohlin model. This theorem is important for understanding the effects of factor growth on the evolution of comparative advantage. The theorem is also important for understanding that large changes in factor supplies, especially in a small country with little influence over world prices, can be absorbed by changes in the composition of output with possibly very small changes in factor prices.

Endnotes

1. Unfortunately, these three assumptions are not quite sufficient. It is possible that the two unit-value isoquants in Figure 8.3 cross twice, creating a situation know as factor-intensity reversal. Ruling this out requires an additional assumption. We will discuss this briefly toward the end of the chapter.