

**Problem Set 9**

1. Consider the following problem facing a firm. The firm has a profit function  $P(k) = Ak_t^\alpha$ , where  $k_t$  is the stock of productive capital, and  $A > 0$  and  $0 < \alpha < 1$ . The firm accumulates capital according to

$$k_{t+1} = x_t + (1 - \delta)k_t,$$

where  $x_t$  is investment and  $0 < \delta < 1$  is the depreciation rate. Investment activities are costly  $C(x_t) = (b/2)x_t^2$ , where  $b > 0$ . The firm solves the following problem:

$$\max \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t [P(k_t) - C(x_t)] dt$$

subject to

$$k_{t+1} = x_t + (1 - \delta)k_t.$$

- a) Write the Bellman equation.
- b) Find and interpret the first-order conditions for a maximum.
- c) Find the steady state and verify that it is a saddlepoint.

2. Find the solution to each of the difference equations:

- a)  $y_{t+1} - 5y_t = 1$  where  $y_0 = 7/4$ .
- b)  $y_{t+2} + y_{t+1} - 2y_t = 12$  where  $y_0 = 4$  and  $y_1 = 5$ .