

Problem Set 7

1. Consider the following problem facing a firm. The firm has a profit function $P(k) = Ak^\alpha$, where k is the stock of productive capital, and $A > 0$ and $0 < \alpha < 1$. The firm accumulates capital according to

$$\dot{k} = x - \delta k,$$

where x is investment and $0 < \delta < 1$ is the depreciation rate. Investment activities are costly $C(x) = (b/2)x^2$, where $b > 0$. The firm solves the following problem:

$$\max \int_0^\infty e^{-rt} [P(k) - C(x)] dt$$

subject to

$$\dot{k} = x - \delta k;$$

$$k(0) = k_0.$$

- a) Write the current value Hamiltonian.
- b) Find and interpret the first-order conditions for a maximum.
- c) To characterize the solution to the system of first-order conditions, draw a phase diagram (in $x - k$ plane) of this system.
- d) Find the steady state and verify that it is a saddlepoint.

2. Find the solution to each of the differential equations:

a) $dy/dt + 2y = 6$ where $y(0) = 10$.

b) $dy/dt + 4y = 0$ where $y(0) = 1$.

c) $d^2y/dt^2 + dy/dt - 2y = -10$ where $y(0) = 12$ and $dy(0)/dt = -2$.