

**Problem Set 6**

1. Consider an overlapping generation economy with production. On the consumption side, consumers live for two periods. They work when young and retire old. They have preferences:

$$U(c_{1t}, c_{2t+1}) = u(c_{1t}) + \beta u(c_{2t+1})$$

where

$$u(c) = \frac{1}{1 - 1/\sigma} c^{1-1/\sigma}.$$

On the production side, competitive firms produce goods with a Cobb-Douglas technology:

$$Y_t = K_t^\alpha L_t^{1-\alpha}$$

where effective labor is  $L_t = A_t N_t$ . This economy is characterized by technical progress and population growth. The growth rate of technology is  $g$  and the growth rate of cohort size is  $n$ .

- a) Derive the intertemporal budget constraint for a typical consumer.
- b) Derive the demand functions and the saving function.
- c) Assuming  $\sigma = 1$ , find the steady capital-labor ratio.
- d) Under what condition is that economy dynamically efficient? ( $\sigma = 1$ ).

2. Consider an overlapping generation economy with production. Consumers work only when young and have preferences:

$$U(c_{1t}, c_{2t+1}) = \ln(c_{1t}) + \beta \ln(c_{2t+1}).$$

Consumers take prices as given. Competitive firms produce goods with a Cobb-Douglas technology:

$$Y_t = K_t^\alpha L_t^{1-\alpha}$$

where effective labor is  $L_t = H_t N_t$ . Firms take price and knowledge  $H$  as given. There is no population growth ( $N_t = N$ ) and no depreciation ( $\delta = 0$ ). In equilibrium, knowledge  $H$  grows proportionately to the aggregate capital stock:  $H_t = K_t$ .

- a) Derive the intertemporal budget constraint for a typical consumer.
- b) Derive the demand functions and the saving function.
- c) Find the growth rate of per capita output.
- d) Show the impact of an increase in  $\alpha$  on the growth rate of output growth.