

**Problem Set 4**

1. Consider the following closed economy. This economy is populated by a representative consumer and a representative firm. The consumer has preferences

$$U(c_1, \ell_1, c_2, \ell_2) = u(c_1, \ell_1) + \beta u(c_2, \ell_2)$$

where  $u(c, \ell) = \ln(c) + \theta \ln(\ell)$ . The firm has a technology

$$y_1 = f(n_1) \quad y_2 = Af(n_2),$$

where  $f(n) = n^\alpha$ .

- a) Write the representative agent's intertemporal budget constraints.
- b) What are the relevant market clearing conditions?
- c) Define a competitive equilibrium allocation and price system.
- d) Solve for a competitive equilibrium.
- e) What is the impact of an increase in  $A$  on output and employment?

2. Consider the following two-period closed economy. The economy is populated by a representative consumer and a representative firm. The consumer has the following preferences:

$$U(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$$

The firm has the following technology:

$$y_i = Ak_i^\alpha \quad i = 1, 2$$

Investment has the standard time-to-build technology, and firm take prices as given. The first period stock of capital is given. The firm maximizes shareholder's wealth.

- a) Assuming that the consumer engages in trade of the firm's ownership shares, solve the consumer's problem.
- b) Solve the firm's problem.
- c) Assuming  $\alpha = \delta = 1$  and imposing market clearing, show the conditions under which this economy grows.
- d) Assuming  $\alpha = \delta = 1$  and imposing market clearing, show that  $q_1 = k_2$ . Also, what is the value of the firm?