Problem Set 3

1. Consider the following closed economy. This economy is populated by a representative consumer that maximizes her utility

$$\max_{c_1,c_2} u(c_1) + \beta u(c_2)$$

subject to her intertemporal budget constraint, where $u(c) = -(1/\alpha)\exp(-\alpha c)$. She receives a perishable (nonstorable) endowment of y_1 in period 1 and y_2 in period 2.

a) Write the representative agent's intertemporal budget constraints.
b) For what values of α is the period utility increasing and concave?
c) Define a competitive equilibrium allocation and price system.
d) Solve for a competitive equilibrium.
e) What is the impact of an increase in α on the real rate of interest? How about an increase in β?

2. Consider an environment with two consumers. The first consumer receives a perishable endowment of widget y in each period. She solves the following problems:

$$\max_{c_1,c_2} u(c_1) + \beta u(c_2)$$

subject to her intertemporal budget constraint, where $u(c) = \ln(c)$. The second consumer receives a perishable endowment of widget y in each period. He solves the following problems:

$$\max_{d_1,d_2} v(d_1) + \beta^* v(d_2).$$

subject to her intertemporal budget constraint, where $v(d) = \ln(d)$.

a) Find the market clearing conditions.
b) Solve for the competitive equilibrium.
c) Show that trade between the two consumers is intertemporally balanced. Why?
d) Show that the first consumer's consumption grows faster than the second consumer when $\beta > \beta^*$. Why?