

Problem Set 13

1. Consider the following consumer's problem. The consumer wishes to maximize her expected lifetime utility:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}$$

where c denotes consumption and $u(c) = c^{1-\alpha}/(1-\alpha)$, $\alpha > 0$. She faces the following budget constraint:

$$A_{t+1} = R_t(A_t - c_t)$$

given A_0 and where A is the amount of wealth and R is stochastic gross interest. Assume that R is iid with $E[R_t^{1-\alpha}] < 1/\beta$.

- Write Bellman's equation for this problem.
- Find and interpret the first-order conditions.
- Show that the optimal policy function takes the form $c_t = (1-\gamma)A_t$.

2. Consider the following RBC model. Preferences are given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \ln(c_t - n_t^\theta) \right\}$$

Technology is

$$y_t = z_t k_t^\alpha n_t^{1-\alpha}$$

Capital accumulation is

$$k_{t+1} = k_t^\delta x_t^{1-\delta}$$

k_0 given.

- Write Bellman's equation for the planner's problem.
- Find and interpret the first-order conditions.
- Find the reduced form solution for n , c , x , and y .
- Are n , c , and x procyclical?