Problem Set 12

1. Consider the following problem facing a firm. The firm has a production function

$$y_t = A_t k_t^\alpha \qquad\qquad 0 < \alpha < 1$$

where k_t is the capital stock and A_t is the level of technology. The firm accumulates capital according to

$$k_{t+1} = x_t + (1-\delta)k_t \qquad\qquad 0 < \delta < 1$$

where x_t is investment. The level of technology is an iid random variable with mean A and variances σ^2. The firm wishes to maximize the expected present value of its profits:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t [A_t k_t^\alpha - x_t] \right\}.$$

a) Write the Bellman equation.
b) Find and interpret the first-order conditions for a maximum.
c) Find the optimal investment policy of the firm.
d) Discuss the impact of an increase in both A and σ on the investment policy.

2. Consider the following linear rational expectations model:

$$m_t - p_t = -\eta i_t \tag{1}$$

$$i_t = r + E_t(p_{t+1}) - p_t \tag{2}$$

$$m_t = \rho m_{t-1} + \epsilon_t \tag{3}$$

where m is the logarithm of the stock of money, p is the logarithm of the price level, i is the nominal interest rate, r is the real interest rate, and E_t denotes the conditional expectation given all information known at period t.

a) Describe all equations using an economic terminology.
b) What is the standard division of exogenous and endogenous variables? Why?
c) Find the reduced-form for all endogenous variables.
d) Discuss the impact of positive innovation to the money supply rule.