Mid-Term 2

Fall 2007

1. Consider an overlapping generation economy with production. Consumers live for two periods: they work when young and retire when old. They have preferences:

$$U(c_t^y, c_{t+1}^o) = \ln(c_t^y) + \beta \ln(c_{t+1}^o).$$

Each young consumer inelastically supplies one unit of labor, so that aggregate labor supply is N_t (the size of the new cohort). Goods are produced by competitive firms with production technology:

$$Y_t = K_t^\alpha N_t^{1-\alpha}.$$

The capital stock evolves according to

$$K_{t+1} = z_t I_t + (1-\delta) K_t.$$

This economy is characterized by investment-specific technical progress $z_{t+1} = (1+g)z_t$ and population growth $N_{t+1} = (1+n)N_t$.

a) At what rate does this economy grow (along the balanced growth path)? What transformation ensures the intensive form $y_t = k_t^\alpha$?

b) Derive the necessary conditions for a competitive equilibrium (the first-order conditions of the consumer's and firm's problem, as well as the relevant market clearing conditions).

c) Construct the phase diagram for this economy. Find the steady state values of y_t and k_t.

d) Show formally the effect of an increase in the rate of investment-specific techical progress g on the steady state value for y_t and the growth rate of per capita output Y_t/N_t along the balanced growth path.

e) After the rise in g, does this economy converge to the new steady state? If so, at what speed?

2. Consider an economy populated by an infinitely lived dynasty with utility

$$\int_p^{\infty} \mathrm{e}^{-\theta t} \ln(C/N) dt$$

where $\theta = \rho - n$, C is consumption, and N is population. Each member of the dynasty inelastically supplies one unit of labor, so that aggregate labor supply is N. Goods are produced by competitive firms with production technology:

$$Y = K^{\alpha} N^{1-\alpha}.$$

The capital stock evolves according to

$$\dot{K} = dK/dt = zI - \delta K.$$

This economy is characterized by investment-specific technical progress $(1/z)dz/dt = \dot{z}/z = g$ and population growth $(1/N)dN/dt = \dot{N}/N = n$.

a) At what rate does this economy grow (along the balanced growth path)? What transformation ensures the intensive form $y = k^{\alpha}$?

b) Solve the planner's problem using a current value Hamiltonian.

c) Construct the phase diagram for this economy. Find the steady state values of y and k.

d) Show formally the effect of an increase in the rate of investment-specific techical progress g on the steady state value for y and the growth rate of per capita output Y/N along the balanced growth path.

e) After the rise in g, does this economy converge to the new steady state? If so, at what speed?