Mid-Term 2

Fall 2006

1. Consider the following problem facing a firm. The firm has a profit function $P(k) = \Gamma k^{\alpha}$, where k is the stock of productive capital, and $\Gamma > 0$ and $0 < \alpha < 1$. The firm accumulates capital according to

$$\dot{k} = x - \delta k,$$

where x is investment and $0 < \delta < 1$ is the depreciation rate. Investment activities are costly $C(x) = (\phi/2)x^2$, where $\phi > 0$. The firm solves the following problem:

$$\max \int_0^{\infty} e^{-rt} \left[P(k) - C(x)\right] dt$$

subject to

$$\dot{k} = x - \delta k;$$

$$k(0) = k_0.$$

Define the price of capital q (Tobin's marginal q), as the multiplier in the current value Hamiltonian.

a) Find and interpret the first-order conditions for a maximum.
b) To characterize the solution to the system of first-order conditions, draw a phase diagram in q - k plane of this system.
c) Find the steady state.
d) Verify that it is a saddlepoint.
e) Formally discuss the effects of an unexpected and permanent reduction of ϕ on the capital stock, investment, and the price of capital.

2. Consider an overlapping generation economy with production. On the consumption side, consumers live for two periods. They work when young and retire old. They have preferences:

$$U(c_t^y, c_{t+1}^o) = u(c_t^y) + \beta u(c_{t+1}^o)$$

where

$$u(c) = \frac{1}{1-1/\sigma} c^{1-1/\sigma}.$$

On the production side, competitive firms produce goods with a Cobb-Douglas technology:

$$Y_t = Z_t K_t^\alpha N_t^{1-\alpha}.$$

This economy is characterized by technical progress. The growth rate of technology Z_t is g. There is no population growth.

a) Derive the goods demand functions and the saving function of the consumer.
b) Is the saving function upward sloping?
c) Assuming $\sigma = 1$, display the difference equation that characterizes this economy on a diagram, and find the steady capital-(effective) labor ratio.
d) Assuming $\sigma = 1$, show that the steady state is stable. At what rate does this economy converge to the steady state?
e) Assuming $\sigma = 1$, formally discuss the effects of an unexpected and permanent increase in g on the capital-(effective) labor ratio. Discuss how this change will affect per capita output and its growth rate.