Mid-Term 2

Fall 2005

1. Consider an overlapping generation endowment economy with storage. Consumers live for two periods. Each young receives an endowment of y_t at time t, while olds get nothing. The endowments grow at rate g. In this country, consumers have preferences:

$$U(c_{1t}, c_{2t+1}) = u(c_t^y) + \beta u(c_{t+1}^o).$$

where

$$u(c) = \frac{1}{1-1/\sigma} c^{1-1/\sigma}.$$

The size of a cohort of young is N_t at time t. The size of a cohort grows at rate n. Each young must save for old age. They have two vehicles to do so. First, they can trade with each other. Second, they can use the storage technology. For this technology, one units of goods stored at t provides δ units of goods at $t+1$.

a) Write the consumer's problem and its first-oder conditions.

b) Solve for the consumer's demand for goods when young and when old. Find the aggregate demand for goods.

c) Find the solution for aggregate national savings.

d) How does an increase in the growth rate of endowments g affect the share of a country's aggregate output devoted to national savings? How about an increase in the growth rate of cohorts n?

e) How does an increase in the return to storage δ affect the share of a country's aggregate output devoted to national savings?

2. Consider the one-sector neoclassical growth model. The infinitely lived representative consumer has preferences

$$\int_{t=0}^{\infty} e^{-\rho t} u(c(t))dt,$$

where $u(c) = c^{1-1\sigma}/(1-1/\sigma)$. Each consumer inelastically supplies one unit of labor each period. There is no population growth. Firms produce final goods with a constant returns to scale technology

$$Y(t) = F\left(K(t), N(t)\right),$$

where $F(K, N) = K^{\alpha}N^{1-\alpha}$. Capital accumulation evolves according to

$$\dot{K}(t) = (1-\tau)I(t) - \delta K(t)$$

where τ measures inefficiencies in the investment sector.

a) Write the current value Hamiltonian of the Planner's problem.

b) Find and interpret the first-order conditions for a maximum.

c) To characterize the equilibrium, draw a phase diagram in the c-k space.

d) Using the phase diagram, describe the impact on consumption and the capital stock of an unexpected permanent increase in τ.

e) How do changes in τ affect the steady state capital/output ratio and the steady state investment/output ratio? Why?