Mid-Term 2

Fall 2004

1. Consider an overlapping generation economy with production and a government. Consumers live for two periods. They work when young and retire old. They have preferences:

$$U(c_{1t}, c_{2t+1}) = u(c_{1t}) + \beta u(c_{2t+1})$$

where

$$u(c) = \frac{1}{1 - 1/\sigma} c^{1-1/\sigma},$$

where $\sigma > 0$. Also, each young inelastically supplies one unit of labor. There is no population or generation growth. Competitive firms produce goods with a Cobb-Douglas technology:

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha},$$

where $A > 0$ and $0 < \alpha < 1$. Capital accumulates as

$$K_{t+1} = I_t + (1 - \delta)K_t,$$

where $0 < \delta < 1$. To fund frivolous expenditures, the government taxes only workers with a labor income tax, but runs a balanced budget. The income tax rate is $0 < \tau < 1$.

a) Derive the consumer's demand functions and the saving function.

b) Derive the difference equation that describes the equilibrium evolution of the capital labor ratio?

c) From now on, assume that $\sigma = 1$. Draw the phase diagram for the difference equation and find the steady state capital labor ratio.

d) How would an unanticipated increase in the income tax rate affect the diagram and the steady state capital labor ratio?

e) After the tax rate increase, at what speed does the capital labor ratio converge to its new steady state?

2. Consider the following one-sector neoclassical economy populated by a representative consumer, a firm, and a government. The economy is populated by a representative consumer with preferences given by

$$\int_0^\infty e^{-\rho t} u(C) dt,$$

where C is consumption and $0 < \rho < 1$ is the subjective discount rate. The instantaneous utility is

$$u(C) = \frac{1}{1 - 1/\sigma} C^{1-1/\sigma},$$

where $\sigma > 0$ is the elasticity of intertemporal substitution. There is no population growth, and the representative consumer inelastically supplies one unit of labor in each period. The firm produces goods with a Cobb-Douglas production technology

$$Y = K^\alpha N^{1-\alpha},$$

where Y is output, K is the capital stock, N is employment, and $0 < \alpha < 1$ is the capital share. Capital accumulates as

$$\dot{K} = I - \delta K,$$

where I is investment and $0 < \delta < 1$ is the depreciation rate. The government decides on expenditures G, and fund these expenditures using a balanced budget:

$$G = T,$$

where T are tax revenues.

a) Assuming that taxes are lump-sum, write the current value Hamiltonian for a (pseudo) Planner's problem. That is, assume that the Planner takes taxes as given.

b) Derive the first-order conditions and solve for consumption growth $\dot{C}/C$.

c) Find the steady state level of consumption, the capital stock, and output.

d) Illustrate the solution to the Planner's problem using a phase diagram in the C and K plane. Intuitively, is the steady state a saddlepoint?

e) How does an unexpected permanent increase in government expenditures affect consumption and output? Illustrate using the phase diagram.