

**Mid-Term 2**

Fall 2003

1. Consider an overlapping generation economy with production. Consumers live for two periods: they work when young and retire when old. They have preferences:

$$U(c_{1t}, c_{2t+1}) = \ln(c_{1t}) + \beta \ln(c_{2t+1}).$$

They inelastically supply one unit of labor, so that aggregate labor supply is  $N_t$ . Goods are produced by competitive firms with production technology:

$$Y_t = (A_t K_t)^\alpha N_t^{1-\alpha}.$$

The capital stock evolves according to

$$K_{t+1} = I_t + (1 - \delta)K_t.$$

This economy is characterized by capital-augmenting technical progress  $A_{t+1} = (1 + g)A_t$  and population growth  $N_{t+1} = (1 + n)N_t$ .

- a) At what rate does this economy grow? What transformation ensures the intensive form  $y = k^\alpha$ ?
- b) Solve the consumer's problem, and find the savings function.
- c) Find the steady state values of  $y$  and  $k$ .
- d) Assume that the economy is initially out of its steady state. Does this economy converge to its steady state? If so, at what speed?

2. Consider the following consumer's problem. The consumer has lifetime utility that exhibits "spirit of capitalism" and is given by

$$\int_0^{\infty} e^{-\rho t} u(c, a) dt,$$

where  $c$  is consumption,  $a$  is the stock of financial assets, and  $0 < \rho < 1$  is the subjective discount rate. The period utility is  $u(c, a) = \ln(c) + \gamma \ln(a)$  for  $\gamma > 0$ . The consumer faces a budget constraint:

$$\dot{a} = ra + y - c,$$

where  $\dot{a} = da/dt$ ,  $y > 0$  is a constant labor income, and  $0 < r < 1$  is the market interest rate. The consumer is initially endowed with  $a(0) = a_0 > 0$  of financial assets. Finally, assume that  $\rho > (1 + \gamma)r$ .

- a) What are the state and control variables for this problem?
- b) Write the current value Hamiltonian and its first-order necessary conditions.
- c) Using these first-order conditions, solve for consumption growth  $\dot{c}$ . Interpret this condition.
- d) Find the steady state level of consumption and the stock of assets.
- e) Illustrate the solution to the consumer's problem using a phase diagram in the  $c$  and  $a$  plane. (Hint: use the solution for consumption growth and the budget constraint, and draw the phase diagram). Intuitively, is the steady state a saddlepoint?
- f) How does an unexpected permanent increase in labor income  $y$  affect steady state consumption and stock of assets?