Mid-Term 1

Fall 2007

1. Consider the following model economy populated by a representative consumer and a representative firm. The representative consumer has lifetime utility

$$\ln(c_1) + \beta \ln(c_2),$$

where c_1 and c_2 are consumption in period 1 and 2, and $0 < \beta < 1$. The consumer inelastically supplies one unit of labor every period. The representative firm produces goods using technology

$$y_1 = k_1^{\alpha} n_1^{1-\alpha},$$

$$y_2 = k_2^{\alpha} n_2^{1-\alpha},$$

where y_t is output, k_t and n_t are the capital and labor input, and $t = 1, 2$. The capital stock evolves according to

$$k_2 = z x_1 + (1 - \delta) k_1,$$

$$k_3 = z x_2 + (1 - \delta) k_2,$$

where x_1 and x_2 are investment in periods 1 and 2, z is the level of productivity in transforming investment into new equipment, $k_3 = 0$, $k_1 > 0$ is given, and $\delta = 1$. Finally, the aggregate resource constraints are

$$c_1 + x_1 = y_1,$$

$$c_2 + x_2 = y_2.$$

a) Define a competitive equilibrium for this production economy.
b) Solve for the competitive equilibrium.
c) Derive the indifference curve and the production possibility frontier. Graph the equilibrium.
d) How is z related to the price of equipment?
e) Consider two economies that have different levels of productivity to produce equipment: $z^L < z^H$. State (and show) which economy has the highest growth rate of consumption and output, and which economy has the highest aggregate savings and investment.

2. This question studies inequality in the (continuous time) Solow growth model. There are two types of consumers. A fraction $\eta^L = N^L/N$ of the population is low skills and a fraction $\eta^H = N^H/N$ is high skills, where $N = N^L + N^H$ is total population and $\eta_L + \eta_H = 1$. Total population grows at rate $(1/N)dN/dt = \dot{N}/N = n$. Each low skills consumer inelastically supplies one unit of labor for wage W^L. They do not save:

$$S^L = 0 \qquad \text{and} \qquad C^L = W^L N^L.$$

Each high skills consumer inelastically supplies one unit of labor, and have a constant savings rate σ:

$$S^H = \sigma\left[W^H N^H + (r+\delta)K\right] \qquad \text{and} \qquad C^H = (1-\sigma)\left[W^H N^H + (r+\delta)K\right].$$

Goods are produced with a constant return to scale technology:

$$Y = AK^{\alpha_K} N_L^{\alpha_L} N_H^{\alpha_H},$$

where $\alpha_K + \alpha_L + \alpha_H = 1$ and $A = [\eta_L^{\alpha_L}\eta_H^{\alpha_H}]^{-1}$. Finally, capital accumulation follows

$$dK/dt = \dot{K} = I - \delta K.$$

a) Find aggregate savings S as a function of output Y.
b) Find the steady state levels for per capita output $y = Y/N$ and the capital labor ratio $k = K/N$. Display the steady state graphically.
c) In the steady state, what is the share of income that goes to low skill workers and the share that goes to high skill workers?
d) For a given share of capital α_K, how does a reduction in the share of low skills workers α_L affect the steady state level of y and k, the growth rate of Y and y, as well as the distribution of income between low skills and high skills workers?
e) At what rate does the economy converge to its new steady state after the reduction in α_L?