**Mid-Term 1**

Fall 2006

**1.** Consider the following AK model economy populated by a representative consumer and a representative firm. The representative consumer has lifetime utility

$$u(c_1) + \beta u(c_2),$$

where $c_1$ and $c_2$ are consumption in period 1 and 2, and $0 < \beta < 1$. The period utility is given by

$$u(c_i) = \left(\frac{1}{1 - 1/\sigma}\right) c_i^{1-1/\sigma},$$

where $i = 1, 2$. The consumer does not work and there is no population growth (population is $N = 1$). The representative firm produces goods using technology

$$y_i = A_i k_i$$

where $y_i$, $k_i$ are output and the capital stock in period $i = 1, 2$. The level of technology is $A_i > 0$. Capital accumulates via

$$k_2 = x_1 + (1 - \delta)k_1,$$

$$k_3 = x_2 + (1 - \delta)k_2,$$

where $x_i$ is investment, $k_3 = 0$, $k_1 > 0$ is given, and $0 < \delta < 1$.

a) Define a competitive equilibrium for this production economy.

b) Solve for the competitive equilibrium.

c) Graph the equilibrium using indifference curves and the production possibility frontier.

d) Under what condition do consumption and output grow (in equilibrium)?

e) How does a fully anticipated increase in the level of technology $A_2$ affect the equilibrium level of the capital stock and output, as well as the growth rate of output?

**2.** This question studies the impact of corruption on growth in the Solow growth model. Consumers save a fixed proportion $0 < \sigma < 1$ of their income. Population grows at rate $n$. The production technology is

$$Y(t) = K(t)^{\alpha} L(t)^{1-\alpha},$$

where $Y$ is output, $K$ is the stock of capital, $L = AN$ is effective labor, $A$ is labor augmenting technical change, $0 < \alpha < 1$, and technology grows at rate $0 < \gamma < 1$. Capital accumulates via

$$\dot{K} = I - \delta K,$$

where $I$ is investment and $0 < \delta < 1$.

a) Find the steady state levels for output $y = Y/L$ and consumption $c = Y/L$, as well as for the capital (effective) labor ratio $k = K/L$. Display the steady state graphically.

b) Find the golden rule levels of ouptput $y$, consumption $c$, and capital $k$. Is this economy over or under capitalized? (An economy is dynamically inefficient if it is overcapitalized.)

c) Corruption reduces the effectiveness of investment. This changes the accumulation equation to

$$\dot{K} = (1 - \tau)I - \delta K,$$

where $0 < \tau < 1$ measures the amount of distortion created by corruption. Find the steady levels of output $y$, consumption $c$, and capital $k$.

d) In this model, does corruption reduce the growth rates of per capita output $Y/N$ and per capita consumption $C/N$? Why?

e) In this model, does corruption reduce the steady state levels of output $y$ and consumption $c$? Why?