Mid-Term 1

Fall 2005

1. Consider the following economy populated by a representative consumer, a representative firm, and a government. The economy is populated by a representative consumer with lifetime utility

$$u(C_1) + \beta u(C_2),$$

where C_1 and C_2 are consumption in period 1 and 2, and $0 < \beta < 1$. The period utility is given by

$$u(C_i) = \left(\frac{1}{1-1/\sigma}\right) C_i^{1-1/\sigma},$$

where $i = 1, 2$. The consumer inelastically supplies one unit of labor, and there is no population growth. The representative firm produces goods using technology

$$Y_i = A_i K_i^{\alpha} N_i^{1-\alpha}$$

where Y_i, K_i are output and the capital stock in period $i = 1, 2$. The level of technology is $A_i > 0$. Capital accumulates via

$$K_2 = I_1 + (1-\delta)K_1,$$

$$K_3 = I_2 + (1-\delta)K_2,$$

where $K_3 = 0$, $K_1 > 0$ is given, and $0 < \delta < 1$. Finally, the government decides on a spending policy G_1 and G_2. To fund its spending, the government levies (non-distortionary) lump-sum taxes from the consumers, and ensures a balanced budget $G_i = T_i$.

a) Derive the firm's (first period) investment function. For what values of α is the investment function downward slopping?

b) Derive the consumer's (first period) savings function. For what values of σ is the savings function upward slopping?

c) How does a fully anticipated increase in the level of government expenditures G_2 affect both the investment and savings function? (For this comparative static experiment, assume that the savings function slopes up). How should the equilibrium interest rate react?

d) How does a fully anticipated increase in the level of technology A_2 affect both the investment and savings function? (For this comparative static experiment, assume that the savings function slopes up). How should the equilibrium interest rate react?

2. Suppose that an economy can be modelled with the Solow growth model. Consumers save a fixed proportion $0 < \sigma < 1$ of their income. Population grows at rate n. The production technology is

$$Y(t) = AK(t)^{\alpha} N(t)^{1-\alpha},$$

where Y is output, K is the stock of capital, L is labor, and $0 < \alpha < 1$. Capital depreciates at rate δ.

a) Find the steady state value of the capital labor ratio $k = K/L$.

b) Find the growth rate of output and consumption. Do they differ from the growth rate of per capita output and per capita consumption?

c) Analyze the effects of an unanticipated and permanent increase in the level of technology A on the steady state per capita output, consumption, and the real interest rate.

d) Analyze the effects of an unanticipated and permanent increase in the capital's share α on the steady state per capita output, consumption, and the real interest rate.

e) At what speed does the capital labor ratio converge to its new steady state in c) and d) above?