Mid-Term 1
Fall 2004

1. Consider the following consumer's problem. The consumer maximizes her utility

$$\max_{c_1,c_2} u(c_1) + \beta u(c_2)$$

subject to her intertemporal budget constraint, where $u(c) = -(1/\alpha)\exp(-\alpha c)$. She receives an income of y_1 in period 1 and y_2 in period 2.

a) For what values of α is the period utility increasing and concave? What is the elasticity of intertemporal substitution and how does it relate to α?
b) Derive the intertemporal budget constraint and find the first-order conditions of the consumer's problem.
c) Find the consumer's demand functions for c_1 and c_2.
d) How does a change in the interest rate affect first period savings?
e) What role does α play in the sensitivity of savings to changes in the interest rate?

2. Consider the following version of the Solow growth model. The economy is populated by consumers with a fixed savings rate $0 < \sigma < 1$. Each consumer inelastically supplies one unit of labor, such that labor supply N coincides with population size. Population grows at rate $0 < n < 1$: $\dot{N}/N = n$. Goods are produced with production technology:

$$Y = K^{\alpha} N^{1-\alpha},$$

where $0 < \alpha < 1$, Y is output, and K is the capital stock. Capital accumulation follows

$$\dot{K} = IZ - \delta K,$$

where $0 < \delta < 1$, I is investment, and Z is the level of investment-specific technology. The level of technology grows at rate $0 < \gamma < 1$: $\dot{Z}/Z = \gamma$. Finally, markets clear.

a) Find the growth rate of output and per capita output along the balanced growth path. Does per capita output grow? Why?
b) What is the necessary transformation to obtain the intensive form of the production function: $y = k^{\alpha}$?
c) Find the steady state equilibrium y^* and k^*. Display this equilibrium graphically.
d) Discuss the impact of an increase in the rate of technical progress γ on the growth rate of per capita output and the steady state value of y.
e) At what speed does the economy converge to its steady state? [HINT: you must solve the linear approximation of the differential equation.]