Mid-Term 1

Fall 2003

1. Consider an environment with two consumers. The first consumer, consumer a, receives a perishable endowment of widget y_1^a in the first period and y_2^a in the second period. She solves the following problem:

$$\max_{c_1^a, c_2^a} u(c_1^a) + \beta u(c_2^a)$$

subject to her intertemporal budget constraint. The second consumer, consumer b, receives a perishable endowment of widget y_1^b in the first period and y_2^b in the second period. He solves the following problem:

$$\max_{c_1^b, c_2^b} u(c_1^b) + \beta u(c_2^b).$$

subject to her intertemporal budget constraint. Finally, neither consumers hold any assets at birth, and both consumers have preferences described by

$$u(c) = \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma}.$$

a) What are the good markets clearing conditions? Also, what are the intertemporal budget constraints for both consumers?
b) Find the aggregate demand function in both periods.
c) Solve for the competitive equilibrium.
d) Show that trade between the two consumers is intertemporally balanced. Why?
e) Show that, if the income of consumer a grows faster than the income of consumer b, consumer a will be a borrower in period 1 and consumer b will be a lender.

2. Consider a two-period production economy with a representative firm and two types of consumers: workers and capitalists. The lifetime utility of a representative worker is

$$U^w = u(c_1^w) + \beta u(c_2^w),$$

where $u(c^w) = ln(c^w)$, where c denotes consumption. The worker has no access to capital markets and inelastically supplies one unit of labor in each period. Accordingly, he faces the sequence of budget constraints:

$$c_1^w = w_1 \qquad \text{and} \qquad c_2^w = w_2,$$

where w is the wage rate.

The lifetime utility of the representative capitalist is

$$U^k = u(c_1^k) + \beta u(c_2^k),$$

where $u(c^k) = \ln(c^k)$, and c is consumption. She has access to capital markets, and faces the budget constraints:

$$c_1^k + k_2 = (1 + r_1)k_1 + \pi_1 \qquad \text{and} \qquad c_2^k = (1 + r_2)k_2 + \pi_2,$$

where the initial capital stock k_1 is given, r is the rental rate of capital, and π is firm profits. In addition, we are assuming that the depreciation rate is unity (100 percent depreciation).

Finally, the representative firm produces goods using technology

$$y_1 = \Gamma k_1^\alpha n_1^{1-\alpha} \qquad \text{and} \qquad y_2 = \Gamma k_2^\alpha n_2^{1-\alpha}.$$

The firm chooses inputs to maximize profits:

$$\pi_1 = y_1 - w_1 n_1 - (1 + r_1)k_1 \qquad \text{and} \qquad \pi_2 = y_2 - w_2 n_2 - (1 + r_2)k_2.$$

a) Write the aggregate resource constraint for each period.
b) Define and solve for a competitive equilibrium.
c) Does the firm generate any profits?
d) Consider a permanent increase in total factor productivity Γ. Formally describe and discuss the impact of this increase on consumption and investment, as well as on the income and welfare of the worker and capitalist.
e) Consider a permanent increase in the capital share α. Formally describe and discuss the impact of this increase on consumption and investment, as well as on the income and welfare of the worker and capitalist.