ECON7020: MACROECONOMIC THEORY I Martin Boileau

Final

Fall 2007

General Instructions

- This examination is 2 1/2 hours in length.
- Answer all questions.

1. Consider the following growth model with investment-specific technical change. The economy is populated by an infinitely-lived representative consumer with preferences:

$$\sum_{j=0}^{\infty} \beta^t \ln(C_t),$$

where C_t denotes consumption. Each consumer inelastically supplies one unit of labor. There is no population growth and the population is normalized to unity. Accordingly, labor supply is $N_t = 1$. Output is produced by a representative firm with a Cobb-Douglas production technology:

$$Y_t = K_t^{\alpha} N_t^{1-\alpha},$$

where Y_t is output and K_t is the capital stock. The capital stock evolves by adding newly produced machinery and equipment to the existing stock. New equipment are produced via a linear production function:

$$X_t = I_t Z_t,$$

where X_t is equipment, I_t is investment, and Z_t represents the state of technology to produce equipment. The capital stock follows the time-to-build technology:

$$K_{t+1} = X_t + (1 - \delta)K_t.$$

Technical change is assumed to grow at constant rate γ:

$$Z_{t+1} = (1 + \gamma)Z_t.$$

a) What is the goods market clearing condition?
b) How is Z_t related to the price of equipment?
c) What is the necessary transformation to obtain the intensive form of the production function $y_t = k_t^{\alpha}$?
d) Find the steady state equilibrium for y_t and k_t.
e) Discuss the impact of an increase in γ on the price of equipment, on the growth rate of output, and on the steady state value of y. Provide an economic intuition for your results.

2. Consider the following problem of costly investment under uncertainty. The firm wishes to maximize the present value of dividends:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t d_t \right\},$$

where E_t is the conditional expectations operator, $0 < r < 1$ is the market (constant) interest rate, and d_t is dividends. Dividends are given by

$$d_t = y_t - x_t - c(x_t),$$

where y_t is output and x_t is investment. Output is produced by a constant returns to scale production function:

$$y_t = \Gamma_t k_t,$$

where $\Gamma_t = 1 + z_t$ is the level of technology and k_t is the capital stock. The random variable z_t evolves as

$$z_t = \rho z_{t-1} + \epsilon_t$$

where $0 < \rho < 1$ and ϵ_t is identically and independently distributed with mean 0 and variance σ^2. The capital stock evolves as

$$k_{t+1} = x_t + (1 - \delta) k_t,$$

where $0 < \delta < 1$ is the constant depreciation rate. Finally, investment is costly. The cost function is

$$c(x_t) = \frac{c}{2} x_t^2.$$

a) Write the dynamic programming problem. Find the first-order conditions of this problem and interpret them.
b) Define q_t to be the price of new capital k_{t+1}. How is q_t related to level of investment?
c) Solve for the value of q_t in terms of state, shocks, and parameters.
d) Do investment x_t and the price of capital q_t covary positively with the level of technology? Why?
e) From the deterministic steady state, describe the impact of a positive productivity shock on the price of capital, investment, and the level of capital.

3. Consider the following consumer's problem. The consumer chooses consumption and savings to maximize lifetime utility given by

$$\sum_{j=0}^{\infty} \beta^t u(c_t),$$

where c_t is consumption. In period t, the consumer purchases an amount b_{t+1} of an asset that pays a constant rate of return r. Finally, in period t, the consumer receives an exogenous labor income y_t.

a) What is the consumer's intertemporal budget constraint?

b) Find and interpret the first-order conditions for a maximum.

c) Assuming that consumers have a consumption smoothing motive, find the consumption function. How does consumption and savings relate to permanent income?

d) Assuming that consumers have preferences $u(c) = \ln(c)$, but no consumption smoothing motive, find the consumption function. How does consumption and savings relate to permanent income?

e) Assuming that consumers have preferences $u(c) = \ln(c)$ and no consumption smoothing motive. How would you modify the problem to ensure that consumption be stationary?