

Final
Fall 2006

General Instructions

- This examination is 2 1/2 hours in length.
 - Answer all questions.
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1. Consider the following two-period consumer's problem. The consumer chooses consumption to maximize her lifetime utility:

$$u(c_1) + \beta u(c_2),$$

where c_1 is consumption in the first period and c_2 is consumption in the second period. The period utility function satisfies $u'(c) > 0$ and $u''(c) < 0$. She inelastically supplies one unit of labor per period, so that her labor income is w_1 in the first period and w_2 in the second period.

- a) Derive the consumer's intertemporal budget constraint.
- b) Formally and intuitively describe the reaction of savings to changes in the interest rate.
- c) Formally and intuitively describe the reaction of savings to transitory changes in labor income. (Hint: analyze a case where $dw_1 = 0$ and $dw_2 > 0$.)
- d) Formally and intuitively describe the reaction of savings to permanent changes in labor income. (Hint: analyze a case where $dw_1 = dw_2 > 0$.)

2. Consider the following economy populated by a representative consumer, a representative firm, and a government. The representative consumer chooses consumption c and hours worked n to maximize lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t + \theta g_t - n_t^\eta),$$

where $0 < \beta < 1$ and $\eta > 0$. The parameter θ takes only 2 possible values. If $\theta = 1$ government expenditures g provide utility to the consumer, but if $\theta = 0$ it does not. The representative firm chooses labor n to maximize profits. Output y is produced with

$$y_t = An_t^\alpha,$$

where $A > 1$ and $0 < \alpha \leq 1$. The government issues bonds and collects tax revenues T to provide an exogenous amount of government expenditures $g \geq 0$ and service its outstanding debt b . Its period budget constraint is

$$g_t + (1 + r_t)b_t = b_{t+1} + T_t.$$

- a) Assume that taxes are levied in a lump-sum manner from the consumer. Does Ricardian equivalence hold for this economy? Why?
- b) Assume that taxes are levied as income tax, $T_t = \tau_t w_t n_t$ where τ_t is the tax rate and w_t is the real wage rate. Does Ricardian equivalence hold for this economy? Why?
- c) Assume that taxes are levied in a lump-sum manner, but create an output distortion:

$$y_t = An_t^\alpha - (\phi/2)T_t^2.$$

Does Ricardian Equivalence hold for this economy? Why?

- d) This question builds on part c), but we modify the economy as follows. The consumer can freely borrow/lend abroad at constant interest rate r , where $\beta(1+r) = 1$. Government expenditures do not provide utility: $\theta = 0$. Assuming a benevolent government, what is the optimal path of taxation? When will the government run fiscal deficits? Why?

3. Consider the following consumer's problem. The consumer chooses consumption and savings to maximize lifetime utility given by

$$\sum_{j=0}^{\infty} \beta^j u(c_j),$$

where c_t is consumption. In period t , the consumer purchases an amount b_{t+1} of an asset that pays a constant rate of return r . Finally, in period t , the consumer receives an exogenous labor income y_t .

- a) What is the consumer's intertemporal budget constraint?
- b) Find and interpret the first-order conditions for a maximum.
- c) Assuming that consumers have a consumption smoothing motive, find the consumption function. How does consumption and savings relate to permanent income?
- d) Assuming that consumers have preferences $u(c) = \ln(c)$, but no consumption smoothing motive, find the consumption function. How does consumption and savings relate to permanent income?