ECON7020: MACROECONOMIC THEORY I Martin Boileau

Final

Fall 2005

General Instructions

- This examination is 2 1/2 hours in length.
- Answer all questions.

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1. Consider the following infinite horizon overlapping generations model with no population growth. Consider the choice of an agent. When young, the agent inelastically supplies one unit of labor for which she receives a wage rate of w_t. The agent also chooses consumption c_{1t} and savings s_t to maximize her expected lifetime utility:

$$\ln(c_{1t}) + \beta E_t \Big\{ \ln(c_{2t+1}) \Big\}.$$

When old, the agent consumes c_{2t+1} financed from her savings. The technology to produce goods is

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha},$$

where k_t is the stock of capital, z_t is the level of technology, and n_t is the labor input. The level of technology evolves as

$$\ln(z_t) = (1-\rho)\ln(z) + \rho \ln(z_{t-1}) + \epsilon_t,$$

where ϵ_t is $iidN(0, \sigma^2)$. Finally, capital accumulation is

$$k_{t+1} = x_t,$$

where x_t is investment.

a) Solve for a competitive equilibrium.

b) Discuss the business cycle implications of this economy (i.e. are consumption and investment procyclical, are they more volatile than output, and so on.)

c) Imagine an alternate economy that is identical, except that it is populated by a representative consumer who supplies labor inelastically and has the following expected lifetime utility:

$$E_0 \Big\{ \sum_{t=0}^{\infty} \beta^t \ln(c_t) \Big\}.$$

Solve for a competitive equilibrium.

d) Discuss the business cycle implications of this alternate economy.

2. Consider the following version of the one-sector neoclassical growth model. The economy is populated by a representative consumer, a firm, and a government. Preferences are

$$\sum_{t=0}^{\infty} \beta^t u(C_t, N_t),$$

where C is consumption, N is hours worked, and $0 < \beta < 1$ is the subjective discount factor. The instantaneous utility is

$$u(C_t, N_t) = \ln(C - (\theta/\eta)N_t^\eta).$$

There is no population growth. The firm produces goods with a Cobb-Douglas production technology

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha},$$

where Y is output, K is the capital stock, N is employment, A is the level of total factor productivity, and $0 < \alpha < 1$ is the capital share. There is no technical progress. Capital accumulates as

$$K_{t+1} = I_t + (1-\delta)K_t,$$

where I is investment and $0 < \delta < 1$ is the depreciation rate. The government balances its budget

$$G_t = T_t,$$

where G is government expenditures and T is lump-sum taxes. Finally, all markets clear. The goods market clearing condition is

$$C_t + I_t + G_t = Y_t.$$

a) Find the necessary first-order conditions of the consumer's and firm's problems.
b) Find the decision rules for output and employment (i.e. find $Y_t = h_y(A_t, K_t)$ and $N = h_n(A_t, K_t)$).
c) How do output and employment react to an unexpected one-period increase in the level of technology A_t? Why?
d) How do output and employment react to an unexpected one-period increase in the level of government expenditures G_t? Why?

3. Consider the following 2-period asset-pricing problem in general equilibrium. The economy is populated by a representative consumer who wishes to solve the following problem:

$$\max E_0 \left\{ \sum_{t=0}^{1} \beta^t \left[c_t - \frac{a}{2} c_t^2 \right] \right\}$$

subject to

$$c_0 + p_0 a_1 + q_0 s_1 = y_0$$

$$c_1 = d_1 a_1 + b_1 s_1.$$

That is, the consumer can buy claims to one-period lived trees. There are two such trees. In period 0, she purchases an amount a_1 at price p_0 of claims to the period 1 stochastic output d_1 of the first tree and an amount s_1 at price q_0 of claims to the period 1 stochastic output b_1 of the second tree.

a) Is the consumer risk averse? Does she have a precautionnary savings motive?

b) Find the pricing equation for both trees.

c) Assume that d_1 is distributed with mean $\bar{d}$ and variance σ^2 and that b_1 is distributed wiht mean $\bar{b} = \bar{d}$ and variance $\eta\sigma^2$, $\eta > 1$. Finally, assume that the correlation between d_1 and b_1 is $Corr(d_1, b_1) = 0$. Show that $p_0 > q_0$. Explain

d) Assume that the second period budget constraint now is

$$c_1 = y_1 + d_1 a_1 + b_1 s_1,$$

such that $c_1 = y_1 + d_1 + b_1$ in equilibrium. Assume that the endowment y_1 is distributed with mean 0 and variance σ^2. Also assume that both d_1 and b_1 are distributed with mean $\bar{d}$ and variance σ^2. Finally, assume that the correlations between y_1, d_1, and b_1 are as follows: $Corr(y_1, d_1) = -1$, $Corr(y_1, b_1) = 0$, and $Corr(d_1, b_1) = 0$. Show that $p_0 > q_0$. Explain.