Final Examination

December 2004

General Instructions

- This examination is 2 1/2 hours in length.
- Answer all questions.

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1. Consider an environment with two consumers. The first consumer, consumer a, receives a perishable endowment of widget y_1^a in the first period and y_2^a in the second period. She solves the following problem:

$$\max_{c_1^a, c_2^a} u(c_1^a) + \beta u(c_2^a)$$

subject to her intertemporal budget constraint. The second consumer, consumer b, receives a perishable endowment of widget y_1^b in the first period and y_2^b in the second period. He solves the following problem:

$$\max_{c_1^b, c_2^b} u(c_1^b) + \beta u(c_2^b).$$

subject to her intertemporal budget constraint. Finally, neither consumers hold any assets at birth, and both consumers have preferences described by

$$u(c) = \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma}.$$

a) What are the good markets clearing conditions? Also, what are the intertemporal budget constraints for both consumers?
b) Find the aggregate demand function in both periods.
c) Solve for the competitive equilibrium.
d) Show that trade between the two consumers is intertemporally balanced. Why?
e) Show that, if the income of consumer a grows faster than the income of consumer b, consumer a will be a borrower in period 1 and consumer b will be a lender.

2. Consider the following growth model with investment-specific technical change. The economy is populated by an infinitely-lived representative consumer with preferences:

$$\sum_{j=0}^{\infty} \beta^t \ln(C_t),$$

where C_t denotes consumption. Each consumer inelastically supplies one unit of labor. There is no population growth and the population is normalized to unity. Accordingly, labor supply is $N_t = 1$. Output is produced by a representative firm with a Cobb-Douglas production technology:

$$Y_t = K_t^{\alpha} N_t^{1-\alpha},$$

where Y_t is output and K_t is the capital stock. The capital stock evolves by adding newly produced machinery and equipment to the existing stock. New equipment are produced via a linear production function:

$$X_t = I_t Z_t,$$

where X_t is equipment, I_t is investment, and Z_t represents the state of technology to produce equipment. The capital stock follows the time-to-build technology:

$$K_{t+1} = X_t + (1-\delta)K_t.$$

Technical change is assumed to grow at constant rate γ:

$$Z_{t+1} = (1+\gamma)Z_t.$$

a) What is the goods market clearing condition?
b) How is Z_t related to the price of equipment?
c) What is the necessary transformation to obtain the intensive form of the production function $y_t = k_t^{\alpha}$?
d) Find the steady state equilibrium for y_t and k_t.
e) Discuss the impact of an increase in γ on the price of equipment, on the growth rate of output, and on the steady state value of y. Provide an economic intuition for your results.

3. Consider the following one-tree pure exchange economy. In this economy, the only consumable good is the fruit (dividend) that grows on the tree. The representative consumer has expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where E_t is the conditional expectation operator, c_t is consumption, β is the subjective discount factor, and $u'(c) > 0$ and $u''(c) < 0$. The representative consumer faces the budget constraint

$$c_t + q_t b_{t+1} + p_t s_{t+1} = y_t + b_t + (p_t + d_t) s_t$$

where q_t and b_t are the price and quantity of a risk-free bond and p_t and s_t are the price and quantity of a risky claim to the stream of dividend d_t. Dividend growth follows the following process

$$d_{t+1}/d_t = 1 + g_{t+1}$$

where $g_{t+1} \approx \ln(1 + g_{t+1})$ is normally distributed with mean γ and variance σ^2.

In equilibrium, the risk-free and risky assets market clearing conditions are $b_t = 0$ and $s_t = 1$.

a) Derive the asset pricing (Euler) equations for both risk-free and risky assets?

b) Theoretically, is the risky rate of return larger than the risk-free rate of return? To answer this question, you must derive the equity risk premium.
Hint 1: $E(xy) = E(x)E(y) + COV(x, y)$.

c) Assuming that $u(c_t) = \ln(c_t)$, solve for the equilibrium price p_t of the risky asset.

d) Assuming that $u(c_t) = \ln(c_t)$, solve for the equilibrium price q_t of the risk-free asset.
Hint 2: $x = \exp(\ln(x))$.
Hint 3: if x is normally distributed with mean μ_x and variance σ_x^2, then $E[\exp(x)] = \exp(\mu_x + \sigma_x^2/2)$.

e) Using your answers to c) and d), is the risky rate of return larger than the risk-free rate of return?