Final Examination

December 2003

General Instructions
- This examination is 2 1/2 hours in length.
- Answer all questions.

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1. Consider the following two-period consumer's problem. The consumer chooses consumption to maximizes her lifetime utility:

$$u(c_1) + \beta u(c_2),$$

where c_1 is consumption in the first period and c_2 is consumption in the second period. The period utility function satisfies $u'(c) > 0$ and $u''(c) < 0$. She inelastically supplies one unit of labor per period, so that her labor income is w_1 in the first period and w_2 in the second period.

a) Derive the consumer's intertemporal budget constraint.
b) Formally and intuitively describe the reaction of savings to changes in the interest rate.
c) Formally and intuitively describe the reaction of savings to transitory changes in labor income. (Hint: analyze a case where $dw_1 = 0$ and $dw_2 > 0$.)
d) Formally and intuitively describe the reaction of savings to permanent changes in labor income. (Hint: analyze a case where $dw_1 = dw_2 > 0$.)
e) Assume that the consumer has a consumption smoothing motive. How does this affect your answers to b), c), and d)?

2. Consider the following 2-period asset-pricing problem in general equilibrium. The economy is populated by a representative consumer who wishes to solve the following problem:

$$\max E_0 \left\{ \sum_{t=1}^{2} \beta^t \left[c_t - \frac{a}{2} c_t^2 \right] \right\}$$

subject to

$$c_1 + pa + qs = y_1$$

$$c_2 = da + bs.$$

That is, the consumer can buy claims to one-period lived trees. There are two such trees. In period 0, she purchases an amount a at price p of claims to the period 1 stochastic output d of the first tree and an amount s at price q of claims to the period 1 stochastic output b of the second tree.

a) Is the consumer risk averse? Why?

b) Find and discuss the pricing equation for both trees.

c) Assume that d is distributed with mean $\bar{d}$ and variance σ^2 and that $b = \gamma d$, $\gamma < 1$. Show that $p > q$. Explain. (Hint: in general equilibrium, $c_2 = d + b$.)

d) Assume that d is distributed with mean $\bar{d}$ and variance σ^2 and that b is distributed with mean $\bar{b} = \bar{d}$ and variance $\eta\sigma^2$, $\eta > 1$. Finally, assume that the correlation between d and b is $Corr(d, b) = 0$. Show that $p > q$. Explain

e) Assume that the second period budget constraint now is

$$c_1 = y_2 + da + bs,$$

such that $c_2 = y_2 + d + b$ in equilibrium. Assume that the endowment y_2 is distributed with mean 0 and variance σ^2. Also assume that both d and b are distributed with mean $\bar{d}$ and variance σ^2. Finally, assume that the correlations between y_1, d, and b are as follows: $Corr(y_2, d) = -1$, $Corr(y_2, b) = 0$, and $Corr(d, b) = 0$. Show that $p > q$. Explain.

3. Consider the following problem of costly investment under uncertainty. The firm wishes to maximize the present value of dividends:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t d_t \right\},$$

where E_t is the conditional expectations operator, $0 < r < 1$ is the market (constant) interest rate, and d_t is dividends. Dividends are given by

$$d_t = y_t - x_t - c(x_t),$$

where y_t is output and x_t is investment. Output is produced by a constant returns to scale production function:

$$y_t = \Gamma_t k_t,$$

where $\Gamma_t = 1 + z_t$ is the level of technology and k_t is the capital stock. The random variable z_t evolves as

$$z_t = \rho z_{t-1} + \epsilon_t$$

where $0 < \rho < 1$ and ϵ_t is identically and independently distributed with mean 0 and variance σ^2. The capital stock evolves as

$$k_{t+1} = x_t + (1 - \delta) k_t,$$

where $0 < \delta < 1$ is the constant depreciation rate. Finally, investment is costly. The cost function is

$$c(x_t) = \frac{c}{2} x_t^2.$$

a) Write the dynamic programming problem. Find the first-order conditions of this problem and interpret them.
b) Define q_t to be the price of new capital k_{t+1}. How is q_t related to level of investment?
c) Solve for the value of q_t in terms of state, shocks, and parameters.
d) Do investment x_t and the price of capital q_t covary positively with the level of technology? Why?
e) From the deterministic steady state, describe the impact of a positive productivity shock on the price of capital, investment, and the level of capital.