“Many branches of both pure and applied mathematics are in great need of computing instruments to break the present stalemate created by the failure of the purely analytical approach to nonlinear problems”

--- John Von Neumann, 1945

Course Objectives

There seems to be an increasing use of simulation techniques in economics. There are several reasons for this. (1) for theoretical modeling, traditional analytical methods have sharp limitations. (2) for empirical analysis, traditional canned software might not be adequate for some questions and there is a huge role for simulations in performing counter-factual experiments on results. (3) many authors effectively use simulations to provide numerical (and then graphical) examples of opaque analytical results.

The objective of this course is to provide an introduction to simulation modeling in microeconomics in order to illustrate all of these advantages. There will be an emphasis on simple general-equilibrium models (which cannot be solved analytically past extremely simple cases) using common and well-known problems in problems in public economics, labor economics, international trade, environmental economics, industrial organization and in other applied micro fields.

But I will also provide a good number of other examples that illustrate the computation of problems in games and information, econometrics (e.g., constrained, non-linear least squares), matrix algebra (inverting a matrix) and calibration (basis for doing structural estimation).

Basic Information

There is no textbook for the course. All course materials will be available on both my personal website and the course website. Readings will be available on my personal web site:

http://spot.colorado.edu/~markusen. Click on “Teaching”, click on “Simulation modeling in microeconomics”.
The course will use the software GAMS (general algebraic modeling system), a demo version which is downloadable for free - and large enough for anything we will be doing. It is already installed on all the machines in the Econ building computer labs. GAMS is widely used by economists and engineers for optimization problems and for solving systems of equations and inequalities (e.g., GAMS is used by engineers for refinery scheduling programs).

You will find GAMS notation and syntax much clearer than that in other packages such as gauss, which is a bad compromise between econometrics and simulation. The GAMS solvers are extremely robust and powerful (meaning handling problems that gauss cannot such as complementarity problems (virtually all general-equilibrium models are complementarity problems)).

I will not provide a detailed tutorial on GAMS notation and syntax. Begin with the link on my website

“Introduction to GAMS Chapter 1"

for downloading instructions and a lot of basic details. Ignore anything about a license file, you don’t need one for the demo version (good up to perhaps 100 equations and unknowns). GAMS is also installed on computers in both our Econ labs.

Note for Apple users: GAMS won’t work on Apple products, which are not really computers and never were. They’re gadgets, and the recent outpouring over Steve Jobs is rather sickening. The solution is to partition your hard drive and then install Windows on one part. That will work and CU ITS presumably has the licenses to allow you to do this for free. The GAMS website has an alternative but very convoluted way of installing and running on MACs but it is awkward and don’t use it. Other real operating systems like UNIX are definitely compatible and discussed on the GAMS website.

Then start on

“Introduction to GAMS Chapter 2"

though this is now old and incomplete and I will be getting back to you on this.

I would like you to download GAMS master all of the material in these two files/chapters files before the first class.

For more detailed information about GAMS, you can consult the GAMS web site: www.gams.com. Click on documentation, and then on GAMS - A User’s Guide. This will give you a lot of the basics you need to know. Unfortunately, this guide is badly out of date and focuses entirely on optimization problems, whereas applied GE modeling generally involves solving square systems of equations and inequalities. But the user’s guide will give you the syntax and notation as I indicated.
After you have gone through the two “Introduction..” files, you can start on the models themselves, which are found by clicking on the bullet “GAMS examples 2012”. The model numbers correspond to chapters and section numbers, so M2-3.gms is the model for Chapter 2, section 3.

Definitely go through M2-1 which is discussed in detail in Introduction Part2. Also go through M2-2 and M2-3 which are unfortunately do not have written chapter text to go with them, but are designed to explain the solves using something you know well: maximizing a Cobb-Douglas utility with two goods and a linear budget constraint.

Assessment for the class will be (A) in the form of exercises, (B) each student will do a project, in which an economic problem is calibrated to a numerical model and simulated. I am quite flexible about project topics: applied general-equilibrium questions, games and information, writing code for econometrics estimation, and so forth. If time permits, I will ask every student to do a short presentation of their project.

Marks for the project will depend a lot on how far away you move from the examples I present. Two important points: (1) You can combine this project with other class or thesis requirements, but you must inform me of this and discuss it with me. (2) You must note and credit ideas or coding borrowed from other programs. To not do so is plagiarism.

Important dates:

Monday February 13: preliminary project proposal due
Monday March 19: first version of project
Thursday May 3: projects due

Jargon: the course will use the following subsystems, referred to as “solvers”

NLP non-linear programming. Optimization subject to non-linear and inequality (e.g., bounded variables) constraints.

MCP mixed complementarity problem. Solves n inequalities in n unknowns (typically bounded, e.g., non-negative variables). “Mixed” refers to equations and inequalities. “Complementarity” refers to an association of each equation/inequality with a particular unknown (necessary in an economic model to compute corner solutions). If an inequality is strict in equilibrium (e.g., a production technique is unprofitable), the associated complementary variable is zero (output from that activity is zero).

MPEC mathematical programming with equilibrium constraints. Combines NLP (optimization) with an MCP as the constraint set (e.g., the constraint set is a general-equilibrium model). Can be used to solve directly for optimal taxes and optimal tariffs for example.
A Basic Course in Economic Simulation Modeling using GAMS

Preface etc.

Chapter 1: Introduction to GAMS for economic problems
(Jesper Jensen’s “Introduction to GAMS Chapter 1” on the website)

Chapter 2: Examples of economic equilibrium problems translated into GAMS
(use “Introduction to GAMS Chapter 2” for now - needs revision)

2.1 Simple supply-demand problem illustrating complementarity
Model M2-1

2.2 Maximization of utility subject to a linear budget constraint
Formulated as a NLP problem:
Formulated as an MCP using first-order conditions:
Formulated as an MCP using Marshallian and Hicksian demand functions:
Model M2-2

2.3 Extension of the utility optimization problem: add a rationing constraint
Formulated as a NLP problem and an MCP.
Formulated as an MPEC
Automating scenario generation
Model M2-3

2.4 Brief introduction to sets: Model M2-3 in set notation
Model M2-4

2.5 Toward general equilibrium: a simple one-good, one-factor, one consumer example
Model M2-5

Chapter 3: The Basic Closed-Economy General-Equilibrium Model as an MCP

3.1 The structure of a general-equilibrium model: optimization at the sector and household level

3.2 Micro-consistent data: product exhaustion and market clearing

3.3 Calibration and replication: background: production, cost and expenditure functions, Shepard’s lemma for the Cobb-Douglas function
3.4 Two goods, two factors, one representative consumer  
Model M3-4a  
Model M3-4b adds taxes

3.5 Initially slack activities  
Model M3-5

3.6 Labor-leisure decision  
Model M3-6

3.7 Two households with different preferences and endowments  
Model M3-7

Chapter 4: Examples of Familiar Industrial-Organization Problems Modeled in GAMS

4.1 Cournot and Bertrand oligopoly with continuous strategies  
Application to strategic trade policy  
Model M4-1

4.2 Nash equilibria with discrete strategies  
Model M4-2

4.3 An insurance problem illustrating moral hazzard and adverse selection  
Model M4-3a modeled as an NLP  
Model M4-3b modeled as an MCP

Chapter 5: Examples of Uses of the NLP Solver in Familiar Economics and Statistics Uses

5.1 OLS as an NLP problem  
Model M5-1

5.2 OLS one step up: constrained non-linear least squares with the NLP solver  
Model M5-2

5.5 Reading and Writing to/from Excel  
Model M5-3

5.3 Balancing a matrix to create micro-consistent data using NLP  
Model M5-4

5.4 Matrix inversion as an MCP  
Model M5-5
Chapter 6: General Equilibrium with Distortionary Taxes, Public Goods, Externalities, Optimal Taxation and Redistribution Policies

6.1 Taxes in the benchmark data
Model M6-1

6.2 Labor supply taxation: introducing equal-yield tax reform
Model M6-2a
Model M6-2b introduces equal yield constraint

6.3 Public consumption goods
Model M6-3

6.4 Optimal provision using a Samuelson rule
Model M6-4

6.5 Public intermediate (infrastructure) good with optimal provision
Model M6-5

6.6 Pollution from production affects utility
Model M6-6a
Model M6-6b uses MPEC to solve for the optimal pollution tax
Model M6-6c uses constraint equation to solve for the optimal pollution tax

6.7 Optimal taxation and redistribution
Model M6-7 adapts M3-7 to an MPEC maximizing social welfare

Chapter 7: Adding Scale Economies and Imperfect Competition to General Equilibrium

7.1 A brief introduction to the CES function - more later

7.2 Monopoly, with fixed costs (increasing returns)
Model M7-2

7.3 Oligopoly: Cournot competition with identical products and free entry
Model M7-3

7.4 Monopolistic-competition I: large group
Model M7-4

7.5 Monopolistic-competition II: small group
Model M7-5
Chapter 8: Open Economy Models for Competitive Economies

8.1 Small open economy
   Model M8-1

8.2 Small open economy: tariffs versus trade costs
   Model M8-2

8.3 Small open economy: calibrating to tariffs in the benchmark
   Model M8-3

8.4 Small open economy: modeling a quota
   Model M8-4a modeled with an endogenous (variable) tax equivalent
   Model M8-4b modeled as supply/demand for licenses

8.5 Large economy and the optimal tariff (rest of world not explicitly modeled)
   Model M8-5

8.6 Two-country Heckscher-Ohlin model: Nash tariffs as an iterative MPEC
   Model M8-6a scalar version
   Model M8-6b same model in set notation

Chapter 9: Open Economy Models for Imperfect Competition and Scale Economies

9.1 A two-country oligopoly model
   Model M9-1

9.2 A two-country monopolistic-competition model
   Model M9-2

9.3 Monopolistic-competition with horizontal multinationals
   Model M9-3

Chapter 10: Basics of Dynamic Modeling:

10.1 Comparative steady-state analysis
    Model M10-1

10.2 Converting an Infinite Horizon Problem to an MCP
    Model M10-2 (currently only available in an MPS/GE format)
Chapter 11: Toward CGE Modeling;

10.1 CES functions and the calibrated-share form

10.2 The MPS/GE subsystem of GAMS

10.3 The Armington assumption

10.4 From an IO Table into GAM

Special Accommodations Policy
If you have specific physical, psychiatric, or learning disabilities and require accommodations, let me know early in the semester so that your needs may be appropriately met. You will need to provide documentation of your disability to the Disability Services Office in Willard 322 (telephone 303-492-8671)