ASEN 5227
Aerospace Math 1
Fall 2004
Homework 7
Assigned 14 OCT, Due 21 OCT

1. Verify the following identities, where $\phi = \phi(\mathbf{r}), \psi = \psi(\mathbf{r}),$ and $\mathbf{a} = \mathbf{a}(\mathbf{r})$:
    a. $\text{curl}(\text{grad}\phi) \equiv \mathbf{0}$
    b. $\text{div}(\text{curl}\mathbf{a}) \equiv 0$
    c. $\text{div}(\text{grad}\phi \times \text{grad}\psi) \equiv 0$
2. Verify the following identities:
    a. $\text{grad}(\phi\psi) \equiv \psi\text{grad}\phi + \phi\text{grad}\psi$
    b. $\text{div}(\phi\mathbf{a}) \equiv \mathbf{a} \cdot \text{grad}\phi + \phi\,\text{div}\mathbf{a}$
    c. $\text{curl}(\phi\mathbf{a}) \equiv \phi\text{curl}\mathbf{a} - \mathbf{a} \times \text{grad}\phi$
3. Show that the vector area of a closed surface is zero, that is,
$$\oint\!\!\oint_S \hat{\mathbf{n}}\, dS \equiv 0$$
4. Show that the volume enclosed by a surface S is
$$\text{volume} = \frac{1}{6}\oint\!\!\oint_S \text{grad}(r^2) \cdot \hat{\mathbf{n}}\, dS$$
or
$$\text{volume} = \frac{1}{3}\oint\!\!\oint_S \mathbf{r} \cdot \hat{\mathbf{n}}\, dS$$
5. Let $\phi(\mathbf{r})$ be a scalar field, show that
$$\iiint_R \nabla^2\phi\, d\tau = \oint\!\!\oint_S \frac{\partial\phi}{\partial n} dS$$
where $\partial\phi / \partial n \equiv \hat{\mathbf{n}} \cdot \text{grad}\phi$ is the derivative of ϕ in the outward direction normal to the surface.