ASEN 5227
Aerospace Math 1
Fall 2004
Homework 6
Assigned 7 OCT, Due 14 OCT

1. Determine the directional derivative of the scalar function $\phi = 2xy + z^2$ in the direction of the vector $\mathbf{s} = \hat{\mathbf{e}}_x + 2\hat{\mathbf{e}}_y + 2\hat{\mathbf{e}}_z$ at the point (1,–1,3).

2. Consider the ellipsoid surface defined by

$$F(x, y, z) \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0.$$

Deduce that the outward normal at a point on this surface is given by

$$\hat{\mathbf{n}} = \frac{\frac{x}{a^2}\hat{\mathbf{e}}_x + \frac{y}{b^2}\hat{\mathbf{e}}_y + \frac{z}{c^2}\hat{\mathbf{e}}_z}{\left(\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}\right)^{1/2}}.$$

corrected

3. Let **r** denote a position vector. Show that:

   a. $\text{grad}(r^n) = nr^{n-2}\mathbf{r}$

   b. $\nabla^2(r^n) = n(n+1)r^{n-2}$

   c. $\text{div}(\mathbf{r}) = 3$

   d. $\text{curl}(f(\mathbf{r})) = \mathbf{0}$, where $f(r)$ is an arbitrary continuous function of r with continuous first derivatives.

4. Let **a** and **b** be continuous vector functions of **r** with continuous first derivatives, and let F and G be continuous scalar functions of **r** with continuous first and second derivatives. Show that:

   a. $\text{div}(\mathbf{a}\times\mathbf{b}) \equiv \mathbf{b}\cdot\text{curl }\mathbf{a} - \mathbf{a}\cdot\text{curl }\mathbf{b}$

   b. $\nabla^2(FG) \equiv F\nabla^2 G + 2\nabla F\cdot\nabla G + G\nabla^2 F$

5. Let **r** be the position vector and **a** an arbitrary constant vector. Show that:

   a. $\text{grad}(\mathbf{r}\cdot\mathbf{a}) = \mathbf{a}$

   b. $\text{div}(\mathbf{r}\times\mathbf{a}) = 0$

   c. $\text{curl}(\mathbf{r}\times\mathbf{a}) = -2\mathbf{a}$

   d. $\text{div}(r\mathbf{a}) = r^{-1}\mathbf{r}\cdot\mathbf{a}$

   e. $curl(r\mathbf{a}) = r^{-1}\mathbf{r}\times\mathbf{a}$