ASEN 5227
Aerospace Math 1
Fall 2004
Homework 5
Assigned 28 SEP, Due 7 OCT

1. The position vector in cylindrical coordinates is given by

$$\mathbf{r} = R\hat{\mathbf{e}}_R + z\hat{\mathbf{e}}_Z$$

Show that Newton's second law of motion

$$\mathbf{V} = \frac{d\mathbf{r}}{dt}$$
$$\mathbf{F} = m\frac{d^2 r}{dt^2}$$

for a point particle of mass *m* can be written in component form in cylindrical coordinates as

$$\begin{aligned}
V_R &= \dot{R} \\
V_\phi &= R\dot{\phi} \\
V_Z &= \dot{Z} \\
F_R &= m(\ddot{R} - R\dot{\phi}^2) \\
F_\phi &= m(R\ddot{\phi} + 2\dot{R}\dot{\phi}) \\
F_Z &= m\ddot{Z}
\end{aligned}$$

2. The position vector in spherical coordinates is given by

$$\mathbf{r} = r\hat{\mathbf{e}}_r$$

Show that the velocity and Newton's second law of motion can be written in component form in spherical coordinates as

$$\begin{aligned}
V_r &= \dot{r} \\
V_\theta &= r\dot{\theta} \\
V_\phi &= r\dot{\phi}\sin\theta \\
F_r &= m(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta) \\
F_\theta &= m(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\cos\theta\sin\theta) \\
F_\phi &= m(r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\phi}\dot{\theta}\cos\theta)
\end{aligned}$$

3. Consider a proton of mass *m* projected into a constant magnetic field given by $\mathbf{B} = \hat{\mathbf{e}}_z$ with an initial velocity $\mathbf{V}(0) = \dot{x}_0\hat{\mathbf{e}}_x + \dot{z}_0\hat{\mathbf{e}}_z$. From elementary physics, the force on the proton is given by the Lorentz force

$$\mathbf{F} = \frac{e}{c}(\mathbf{V}\times\mathbf{B}) \quad \text{(Gaussian cgs units)}$$

where e is the charge on the proton and c is the speed of light.

a. By means of Newton's second law of motion, find the differential equations that the Cartesian components of the position vector $\mathbf{r} = x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y + z\hat{\mathbf{e}}_z$ must satisfy.

b. Show that a solution to these equations that satisfies the velocity initial condition is

$$x = \frac{\dot{x}_0}{\omega}\sin\omega t, \quad y = \frac{\dot{x}_0}{\omega}\cos\omega t, \quad z = \dot{z}_0 t$$

where $\omega = eB_z / mc$. Show that this solution describes a circular helix in space.