$$\vec{e}_1 = \frac{\sqrt{3}}{4}\hat{i} + \frac{1}{4}\hat{j}$$

$$\vec{e}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j}$$

$$\vec{e}_3 = \hat{k}$$

a.) Dual basis: $\vec{e}^{\,k} = \dfrac{\vec{e}_i \times \vec{e}_j}{[\vec{e}_1\vec{e}_2\vec{e}_3]}$

$$[\vec{e}_1\vec{e}_2\vec{e}_3] = \vec{e}_1 \cdot (\vec{e}_2 \times \vec{e}_3)$$

$$= \left(\frac{\sqrt{3}}{4}, \frac{1}{4}, 0\right)\cdot\left(\frac{3}{2}, -\frac{1}{2}, 0\right)$$

$$= \frac{3\sqrt{3}}{8} - \frac{1}{8} = \frac{1}{8}(3\sqrt{3}-1)$$

$$\vec{e}^{\,1} = \frac{\vec{e}_2 \times \vec{e}_3}{[\vec{e}_1\vec{e}_2\vec{e}_3]} = \frac{\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}}{\frac{1}{8}(3\sqrt{3}-1)} = \frac{4}{3\sqrt{3}-1}(3\hat{i} - \hat{j}) \longleftarrow$$

$$\vec{e}^{\,2} = \frac{\vec{e}_3 \times \vec{e}_1}{[\vec{e}_1\vec{e}_2\vec{e}_3]} = \frac{-\frac{1}{4}\hat{i} + \frac{\sqrt{3}}{4}\hat{j}}{\frac{1}{8}(3\sqrt{3}-1)} = \frac{2}{3\sqrt{3}-1}(-\hat{i} + \sqrt{3}\hat{j}) \longleftarrow$$

$$\vec{e}^{\,3} = \frac{\vec{e}_1 \times \vec{e}_2}{[\vec{e}_1\vec{e}_2\vec{e}_3]} = \frac{\left(\frac{3\sqrt{3}}{8} - \frac{1}{8}\right)\hat{k}}{\frac{1}{8}(3\sqrt{3}-1)} = \hat{k} \longleftarrow$$

b.) $|\vec{e}_i|, |\vec{e}^{\,j}|$

$$|\vec{e}_1| = \frac{1}{4}\left((\sqrt{3})^2 + 1\right)^{\frac{1}{2}} = \frac{1}{4}\sqrt{4} = \frac{1}{2}$$

$$|\vec{e}_2| = \frac{1}{2}(1+9)^{\frac{1}{2}} = \frac{\sqrt{10}}{2}$$

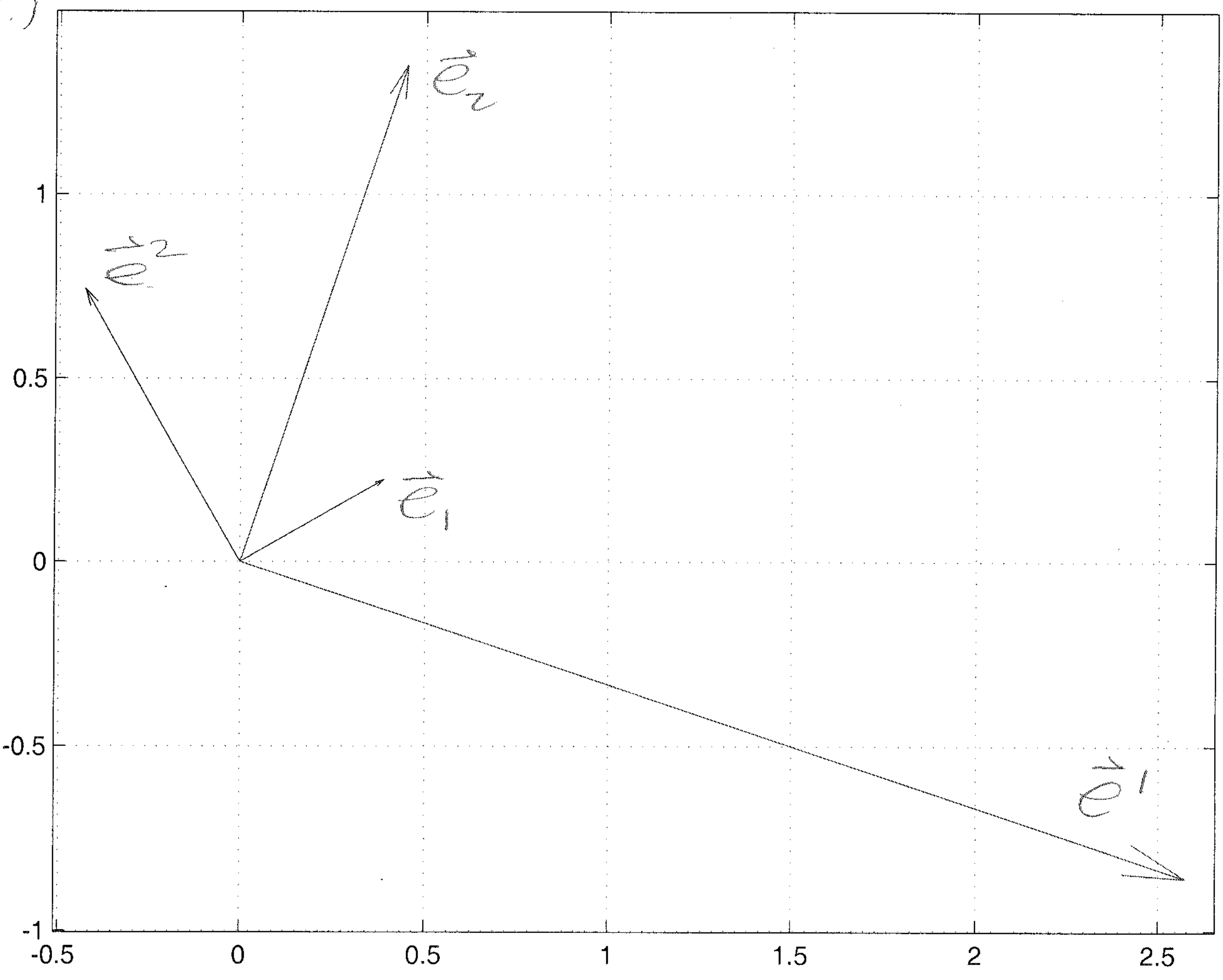
$$|\vec{e}_3| = 1$$

$$|\vec{e}^1| = \frac{4}{3\sqrt{3}-1}(9+1)^{\frac{1}{2}} = \frac{4\sqrt{10}}{3\sqrt{3}-1}$$

$$|\vec{e}^2| = \frac{2}{3\sqrt{3}-1}(1+3)^{\frac{1}{2}} = \frac{4}{3\sqrt{3}-1}$$

$$|\vec{e}^3| = 1$$

C.)



Continuing Prob 6.

$$a^1 = 1, \quad a^2 = 2, \quad a^3 = 3$$

$$\rightarrow \quad \vec{a} = \vec{e}_1 + 2\vec{e}_2 + 3\vec{e}_3$$

From notes Eq. (2) p. 19:

$$a_i = \vec{a} \cdot \vec{e}_i$$

$$a_1 = (\vec{e}_1 + 2\vec{e}_2 + 3\vec{e}_3) \cdot \vec{e}_1$$

$$= \left[\left(\frac{\sqrt{3}}{4} + 2\left(\frac{1}{2}\right)\right)\hat{i} + \left(\frac{1}{4} + 2\left(\frac{3}{2}\right)\right)\hat{j} + 3\hat{k}\right] \cdot \left(\frac{\sqrt{3}}{4}\hat{i} + \frac{1}{4}\hat{j}\right)$$

$$= \left(\frac{\sqrt{3}}{4} + 1\right)\left(\frac{\sqrt{3}}{4}\right) + \left(\frac{1}{4} + 3\right)\left(\frac{1}{4}\right) = \frac{3}{16} + \frac{\sqrt{3}}{4} + \frac{1}{16} + \frac{3}{4}$$

$$= 1 + \frac{\sqrt{3}}{4} \longleftarrow$$

$$a_2 = \left[\left(\frac{\sqrt{3}}{4} + 1\right)\hat{i} + \frac{13}{4}\hat{j} + 3\hat{k}\right] \cdot \left(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j}\right)$$

$$= \frac{\sqrt{3}}{8} + \frac{1}{2} + \frac{39}{8} = \frac{39\sqrt{3}}{8} + \frac{1}{2} \longleftarrow$$

$$a_3 = \left[\left(\frac{\sqrt{3}}{4} + 1\right)\hat{i} + \frac{13}{4}\hat{j} + 3\hat{k}\right] \cdot \hat{k} = 3 \longleftarrow$$

Of course this is done much easier writing as a linear transformation, as we do later.