ASEN 5227
Aerospace Math 1
Fall 2004
Homework 2
Assigned 2 SEP, Due 9 SEP

1. Verify the following identities
    a. $\delta_{ii} = 3$
    b. $\delta_{ij}\delta_{ij} = \delta_{ii}$
    c. $\delta_{ij}\delta_{jk} = \delta_{jk}$
    d. $\varepsilon_{ijk}\varepsilon_{ijk} = 6$
    e. $a_i a_j \varepsilon_{ijk} = 0$
2. Prove the ε–δ identity using the identity for the vector triple product.

3. Prove the following vector identity in an orthonormal coordinate system using index notation:

$$(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d}) = [\mathbf{a}\cdot(\mathbf{c}\times\mathbf{d})]\mathbf{b} - [\mathbf{b}\cdot(\mathbf{c}\times\mathbf{d})]\mathbf{a}\ .$$

4. In a rectangular Cartesian coordinate system find the length and direction cosines of a vector **a** that extends from the point (1,–1,3) to the midpoint of the line segment from the origin to the point (6,–6,4).
5. The vectors **a** and **b** are defined as

$$\mathbf{a} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{k}}$$
$$\mathbf{b} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

   where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are an orthonormal basis.
    a. Find the orthogonal projection of **a** on **b**.
    b. Find the angle between the positive directions of the vectors.
6. Le the vectors $\{\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}\}$ constitute an orthonormal basis. In terms of this basis define a cogredient basis by

$$\mathbf{e}_1 = \frac{\sqrt{3}}{4}\hat{\mathbf{i}} + \frac{1}{4}\hat{\mathbf{j}}$$
$$\mathbf{e}_2 = \frac{1}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}}$$
$$\mathbf{e}_3 = \hat{\mathbf{k}}$$

a. Determine the dual or reciprocal (contragredient basis $\{\mathbf{e}^1, \mathbf{e}^2, \mathbf{e}^3\}$ in terms of the orthonormal basis $\{\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}\}$.

b. Determine the magnitudes (or norms) $|\mathbf{e}_i|$ and $|\mathbf{e}^j|$.

c. Plot the basis vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}^1,$ and $\mathbf{e}^2$.

7. Corresponding to the cogredient basis defined in Exercise 6., let the contragredient components of a vector **a** be given by $a^1 = 1$, $a^2 = 2$, $a^3 = 3$. What are the values of the cogredient components a_1, a_2, and a_3?
8. Construct an orthonormal basis from the set of cogredient vectors in Exercise 6.
9. AEM: P2.35 MATLAB Gram-Schmidt process.

<u>Submission instructions for MATLAB code for Prob. 9</u>

1. Only submit a function m-file. Do not submit a script.
2. Use the file name convention: lastname_firstinitial.m, for example argrow_b.m
3. I will execute by calling: argrow_b(x1,x2,x3)
   where x1, x2, and x3 are the initial vectors from which the Gram-Schmidt process will create an orthonormal set. The function should return the orthogonal vector set.
4. email your m-file as a text file attachment to me at brian.argrow@colorado.edu. The timestamp on the email should be no later than the 0930 class start time.