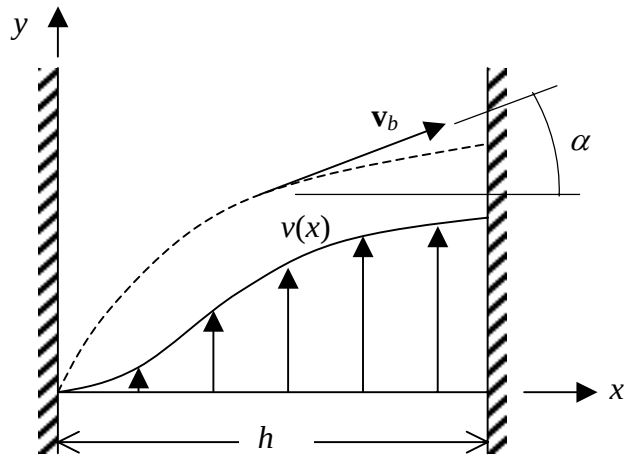
ASEN 5227
Aerospace Math 1
Fall 2004
Homework 12
Assigned 2 Dec, Due 9 DEC

1. P6.7
2. P6.8
3. Using the Lagrange multiplier method, find the critical points of

$$f(x,y,z) = x^2 + y^2 + z^2 - 4z + 16, \quad G(x,z) = x^2 - z = 0.$$

4. Consider the problem of crossing a river in a boat with a constant speed in the shortest possible time. Assume that the banks of the river are straight and parallel. Take the *y* axis to coincide with one of the banks and the *x* axis across the river. The velocity vector $\mathbf{v} = (u, v)$ at any point (*x*,*y*) in the system is given by

$$u = 0, \quad v = v(x)$$



Formulate the problem variationally by writing the time required to cross the river. Follow the procedure given below.

a.) Assume the speed (constant) of the boat is v_b, and use the point (0,0) as point of departure. Let α be the angle that depends on the course of the boat.

b.) Write the actual velocity of the boat in the river:

$$\frac{dx}{dt} = v_b \cos\alpha, \quad \frac{dy}{dt} = v(x) + v_b \sin\alpha.$$

Express $\cos\alpha$ in terms of dy/dx [$y = y(x)$ is the path on which the boat moves] by solving the equation

$$y' = \frac{v \pm v_b \sqrt{1 - \cos^2 \alpha}}{v_b \cos \alpha}.$$

c.) Obtain the time functional in the form

$$t(y) = \int_0^h \frac{\sqrt{v_b^2[1 + (y')^2 - v^2} - vy'}{v_b^2 - v^2} dx, \quad y(0) = 0$$

where $v(x)$ is a given function (stream velocity) of x.

5. Calculate the first variation of: $I(y) = \int_0^1 \sqrt{1 + (y')^2}\, dx$.

6. Calculate the first variation of: $I(y) = \int_0^1 \sqrt{\frac{1 + (y')^2}{y}}\, dx$.