ASEN 5227
Aerospace Math 1
Fall 2004
Homework 10
Assigned 4 NOV, Due 11 NOV

1. Euler's vector equation of motion for a rigid body is

$$\overset{\leftrightarrow}{\mathcal{I}} \cdot \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\overset{\leftrightarrow}{\mathcal{I}} \cdot \boldsymbol{\omega}) = \mathbf{M}.$$

   Introduce Cartesian components such that

$$\overset{\leftrightarrow}{\mathcal{I}} = I_{ij}\hat{\mathbf{e}}_i\hat{\mathbf{e}}_j, \quad \boldsymbol{\omega} = \omega_i\hat{\mathbf{e}}_i, \quad \mathbf{M} = M_i\hat{\mathbf{e}}_i.$$

   Show that Euler's equation can be written in Cartesian tensor form as

$$I_{ij}\dot{\omega}_j + \omega_l\omega_n I_{mn}\varepsilon_{lmi} = M_i \quad i = 1, 2, 3.$$

2. When the body-fixed Cartesian coordinate are aligned with the *principal axes*, the moment-of-inertia tensor takes a simpler form

$$\overset{\leftrightarrow}{\mathcal{I}} = I_{11}\hat{\mathbf{e}}_1\hat{\mathbf{e}}_1 + I_{22}\hat{\mathbf{e}}_2\hat{\mathbf{e}}_2 + I_{33}\hat{\mathbf{e}}_3\hat{\mathbf{e}}_3.$$

   Show that the Euler equations now can be written as

$$\begin{aligned} I_{11}\dot{\omega}_1 + \omega_2\omega_3(I_{33} - I_{22}) &= M_1 \\ I_{22}\dot{\omega}_2 + \omega_1\omega_3(I_{11} - I_{33}) &= M_2 \\ I_{33}\dot{\omega}_3 + \omega_1\omega_2(I_{22} - I_{11}) &= M_3. \end{aligned}$$

3. Find the eigenvalues and eigenvectors (unit) of

   a. $\begin{bmatrix} 4 & -4 & 0 \\ -4 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

   b. $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

   c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

   d. $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

   e. $\begin{bmatrix} 3 & 5 & 8 \\ 5 & 1 & 0 \\ 8 & 0 & 2 \end{bmatrix}$

   f. $\begin{bmatrix} 2 & -\sqrt{3} & 0 \\ -\sqrt{3} & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

   First write the characteristic equation, which you can solve using MATLAB.

4. The components of a stress dyadic at a point, referred to as (X_1, X_2, X_3) system, are (in kips ≡ 1000 psi):

$$\begin{bmatrix} 12 & 9 & 0 \\ 9 & -12 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Find the following

a. The stress vector acting on a plane perpendicular to the vector $2\hat{\mathbf{e}}_1 - 2\hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_3$ passing through the point.

b. The magnitude of the stress vector and the angle between the stress vector and the normal to the plane.

c. The magnitudes of the normal and tangential components of the stress vector.

5. Verify that

$$\overleftrightarrow{\mathbf{I}} \cdot \overleftrightarrow{\mathbf{\Phi}} = \overleftrightarrow{\mathbf{\Phi}} \cdot \overleftrightarrow{\mathbf{I}} = \overleftrightarrow{\mathbf{\Phi}}$$

6. If **a** is an arbitrary vector and $\overleftrightarrow{\mathbf{\Phi}}$ is an arbitrary dyadic, verify that:

a. $(\overleftrightarrow{\mathbf{I}} \times \mathbf{a}) \cdot \overleftrightarrow{\mathbf{\Phi}} = \mathbf{a} \times \overleftrightarrow{\mathbf{\Phi}}$.

b. $(\mathbf{a} \times \overleftrightarrow{\mathbf{I}}) \cdot \overleftrightarrow{\mathbf{\Phi}} = \mathbf{a} \times \overleftrightarrow{\mathbf{\Phi}}$.

c. $(\overleftrightarrow{\mathbf{\Phi}} \times \mathbf{a})^{\mathrm{T}} = -\mathbf{a} \times \overleftrightarrow{\mathbf{\Phi}}^{\mathrm{T}}$.