

**ASEN 5227**  
**AEROSPACE MATHEMATICS 1, FALL 2004**

**Vector Terms and Operations in Cartesian Component Form (from Reddy & Rasmussen).**

Term or Operation	Vector Form	Cartesian Component Form
1. Vector	$\mathbf{A}$	$A_i$
2. Scalar (dot) product	$\mathbf{A} \cdot \mathbf{B}$	$A_i B_i$
3. Vector (cross) product	$\mathbf{A} \times \mathbf{B}$	$\epsilon_{ijk} A_j B_k$
4. Scalar triple product	$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$	$\epsilon_{ijk} A_i B_j C_k$
5. Vector triple product	$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$	$\epsilon_{ijk} \epsilon_{klm} A_j B_l C_m$
6. Divergence of $\mathbf{A}$	$\nabla \cdot \mathbf{A}$	$\frac{\partial A_i}{\partial x_i}$
7. Divergence of $U\mathbf{A}$ ( $U$ , scalar function)	$\nabla U \cdot \mathbf{A} + U \nabla \cdot \mathbf{A}$	$U \frac{\partial A_i}{\partial x_i} + A_i \frac{\partial U}{\partial x_i}$
8. Divergence of $(\mathbf{A} + \mathbf{B})$	$\nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$	$\frac{\partial A_i}{\partial x_i} + \frac{\partial B_i}{\partial x_i}$
9. Divergence of $(\mathbf{A} \times \mathbf{B})$	$\mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$	$\frac{\partial}{\partial x_i} (\epsilon_{ijk} A_j B_k)$
10. Curl of $\mathbf{A}$	$\nabla \times \mathbf{A}$	$\epsilon_{ijk} \frac{\partial A_k}{\partial x_j}$
11. Curl of $U\mathbf{A}$	$\nabla U \times \mathbf{A} + U \nabla \times \mathbf{A}$	$\epsilon_{ijk} U \frac{\partial A_k}{\partial x_j} + \epsilon_{ijk} A_k \frac{\partial U}{\partial x_j}$
12. Curl of $(\mathbf{A} + \mathbf{B})$	$\nabla \times \mathbf{A} + \nabla \times \mathbf{B}$	$\epsilon_{ijk} \frac{\partial A_k}{\partial x_j} + \epsilon_{ijk} \frac{\partial B_k}{\partial x_j}$
13. Curl of $(\mathbf{A} \times \mathbf{B})$	$\mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$	$-\frac{\partial}{\partial x_i} (\epsilon_{ijk} \epsilon_{klm} A_l B_m)$
14. (Curl of $\mathbf{A}) \times \mathbf{B}$	$(\nabla \times \mathbf{A}) \times \mathbf{B} = \mathbf{B} \cdot [\nabla \mathbf{A} - (\nabla \mathbf{A})^T]$	$B_j (\frac{\partial A_i}{\partial x_j} - \frac{\partial A_j}{\partial x_i})$
15. $\nabla^2 U$	$\nabla^2 U = \nabla \cdot (\nabla U)$	$\frac{\partial}{\partial x_i} \frac{\partial}{\partial x_i} U$
16. $\nabla^2 \mathbf{A}$	$\nabla^2 \mathbf{A} = (\nabla \cdot \nabla)\mathbf{A}$	$\frac{\partial}{\partial x_i} \frac{\partial}{\partial x_i} A_j$
17. $\nabla \times (\nabla \times \mathbf{A})$	$\nabla(\nabla \cdot \mathbf{A}) - (\nabla \cdot \nabla)\mathbf{A}$	$\frac{\partial}{\partial x_j} \frac{\partial}{\partial x_i} A_j - \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_j} A_i$
18. $(\mathbf{A} \cdot \nabla)\mathbf{B}$	$(\mathbf{A} \cdot \nabla)\mathbf{B}$	$A_j \frac{\partial B_i}{\partial x_j}$
19. $\mathbf{A}(\nabla \cdot \mathbf{B})$	$\mathbf{A}(\nabla \cdot \mathbf{B})$	$A_i \frac{\partial B_j}{\partial x_j}$