ASEN 5227
Unit Quiz 2
1 Nov

Name________________________________
(Last, First)

Label each of the following statements either true (T) or false (F). A correct response is awarded +1, and incorrect response –1/2, and no response 0. Ambiguous responses are assumed incorrect.

___ 1. For $y = y(t)$

$$\frac{d^2y}{dt^2} + t\frac{dy}{dt} + 5y(t) = e^t \qquad \frac{d^2y}{dt^2} + \sin y = 0$$

the first equation is of 2nd order, and there is a nonzero forcing function on the RHS, this equation is a 2nd-order, nonlinear, nonhomogenous, ordinary differential equation (ODE). It is also nonlinear since the independent variable *t* multiplies a derivative of the dependent variable *y*. The second equation is a 2nd-order, nolinear, homogeneous ODE.

___ 2. A partial differential equation is one that involves the derivatives of at least one dependent variable, and more than one independent variable.

___ 3. Consider the first-order, ordinary differential equation for an *initial-value problem*

$$\frac{dy(t)}{dt} = f(t,y), \quad y(t_0) = y_0 .$$

If the function $f(t,y)$ and its partial derivative $\partial f(t,y)/\partial y$ is continuous in a rectangular region *R* of the *ty*-plane, then an infinite number of solutions exist for the initial-value problem in the region. This region where the function is defined is called the *domain of definition* (or *domain*) of $f(t,y)$. The solution $y(t)$ is the *unique* solution in the domain that passes through the point (t_0, y_0). Radioactive half life is an example of a physical phenomenon governed by a 1st-order ODE.

___ 4. There are *n* linearly independent solutions, $y_1(t),\cdots y_n(t)$ for the nth-order, homogeneous linear ODE $\mathcal{L}_n[y(t)] = 0$ with constant coefficients in the interval [a,b], and with the operator $\mathcal{L}_n$ defined by

$$\mathcal{L}_n \equiv \frac{d^n}{dt^n} + a_{n-1}(t)\frac{d^{n-1}}{dt^{n-1}} + \cdots + a_0(t) .$$

The *general solution* in the interval is a linear combination of the $y_1(t),\cdots y_n(t)$ independent solutions. The general solution to the nonhomogeneous ODE $\mathcal{L}_n[y(t)] = f(t)$ is the sum of the general solution to the homogenous equation $y_c(t)$ and the *particular solution* $y_p(t)$ of the nonhomogeneous equation. The complete solution of $\mathcal{L}_n[y(t)] = f(t)$ is $y(t) = y_p + y_c$.

___ 5. The solution set of $\mathcal{L}_n[y(t)] = 0$ forms an *n*-dimensional vector space, so linear combinations of solutions are also solutions; this is the *principle of superposition*.

6. Write the second-order ODE

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$$

in matrix format as two first order ODE

7. The acceleration of a particle is given as $a(t) = a_0$, were a_0 is a constant. For $t \geq 0$, sketch the acceleration $a(t)$, speed $v(t)$, and position $x(t)$