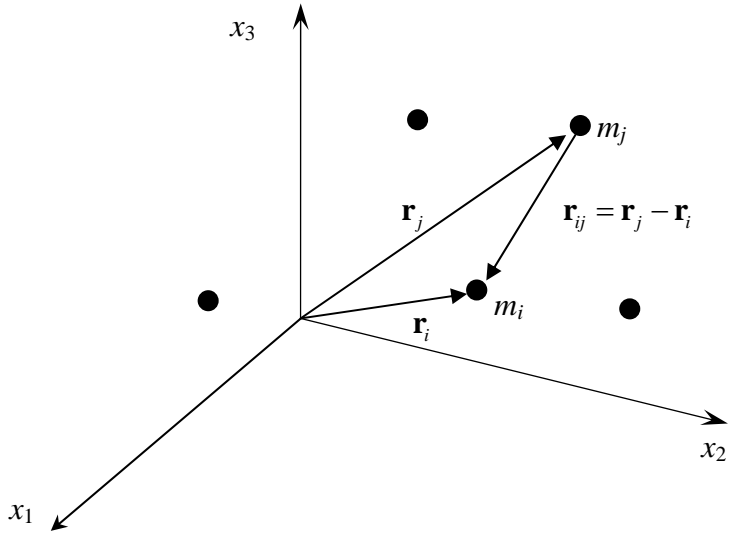


**ASEN 5227
Aerospace Math
Fall 2007**

Project 2: Numerical Integration of the n -Body Problem (v. 1)

Assigned 27 Nov, Due 13 Dec (0930 am)



The n -body problem describes the dynamics of a collection of bodies of with forces between them. Using Newton's law of gravitation, the force \mathbf{f}_i on mass m_i from the other $n - 1$ masses is

$$\mathbf{f}_i = G \sum_{\substack{j=1 \\ j \neq i}}^n \frac{m_i m_j}{r_{ij}^3} (\mathbf{r}_j - \mathbf{r}_i),$$

where $G (= 6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2)$ is the universal gravitation constant the position \mathbf{r}_i , velocity \mathbf{v}_i , of mass m_i , and the distance between m_i and m_j , r_{ij} are given by

$$\mathbf{r}_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{bmatrix}, \quad \dot{\mathbf{r}}_i = \begin{bmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{bmatrix}, \quad r_{ij} = |\mathbf{r}_j - \mathbf{r}_i|.$$

Newton's second law then gives the n vector differential equations:

$$\ddot{\mathbf{r}}_i = G \sum_{\substack{j=1 \\ j \neq i}}^n \frac{m_j}{r_{ij}^3} (\mathbf{r}_j - \mathbf{r}_i).$$

With given initial conditions these equations provide a complete mathematical description of the motion of the system of n mass particles. Note that this represents a system of $6n$ first-order differential equations that must be integrated.

Tasks

1. For the case $n=3$ make the variable substitution:

$$\mathbf{y}_1 = \mathbf{r}_1$$

$$\mathbf{y}_2 = \mathbf{r}_2$$

$$\mathbf{y}_3 = \mathbf{r}_3$$

$$\mathbf{y}_4 = \dot{\mathbf{r}}_1$$

$$\mathbf{y}_5 = \dot{\mathbf{r}}_2$$

$$\mathbf{y}_6 = \dot{\mathbf{r}}_3$$

and write the $6n$ system of first-order equations to be integrated.

2. Write a MATLAB code (I suggest you use ODE45) to simulate n -body motion and explore the behavior of different “universes” of particles with equal masses for $n = 2, 3$,
4. Include and discuss trajectory plots.
3. Can you model the Sun, Earth, Luna system (with average values for the orbital radii)?

Use the AIAA conference and meetings paper template at

<http://www.aiaa.org/content.cfm?pageid=396> and submit your report in pdf format. The document should contain a brief description of your approach (e.g., definitions and equations) and results. You will be graded on the organization and readability of your report, as well as the accuracy of the results. Limit your text length to 6 pages (including figures). Use the file name format: lastname_initial.pdf.

Submit a copy of your MATLAB function or script file. Your code must be well documented.