ASEN 5227
Aerospace Math 1
Fall 2007
Project 1: Coordinate Transformations for Unsteady Frames
Assigned 27 Sep, Due 16 Oct (EOB) **v01, updated 10/7/2007**

Part 1: Frenet Frame[1]

The Frenet frame is a set of coordinates that move with the trajectory of a particle and measure the turning and the twisting that the particle path experiences. This device is a particularly useful tool in orbital mechanics, where the resultant forces are often described in this coordinate system.

Let $\mathbf{r}$ be the position vector defining a curve C in $\mathbf{R}^3$. The vector $\mathbf{r}'$ then defines a tangent to this curve, while $\mathbf{r}''$ is its acceleration vector. The *Frenet* frame is a set of basis vectors constructed from the tangent and the acceleration vectors. First we define a unit tangent vector $\hat{\mathbf{t}}$ by

$$\hat{\mathbf{t}} = \frac{\mathbf{r}'}{\|\mathbf{r}'\|}. \quad (1)$$

Next we define a *binormal* vector $\hat{\mathbf{b}}$ by

$$\hat{\mathbf{b}} = \frac{\mathbf{r}' \times \mathbf{r}''}{\|\mathbf{r}' \times \mathbf{r}''\|}. \quad (2)$$

The *normal* vector $\hat{\mathbf{n}}$ is defined by

$$\hat{\mathbf{n}} = \hat{\mathbf{b}} \times \hat{\mathbf{t}}. \quad (3)$$

Let $s = \|\mathbf{r}'\|$ be the speed of the particle. We define the *curvature*, κ and the *torsion* τ of the space path as

$$\kappa = \frac{\|\mathbf{r}' \times \mathbf{r}''\|}{s^3}, \quad \tau = \frac{(\mathbf{r}' \times \mathbf{r}'') \cdot \mathbf{r}'''}{\|\mathbf{r}' \times \mathbf{r}''\|^2}. \quad (5)$$

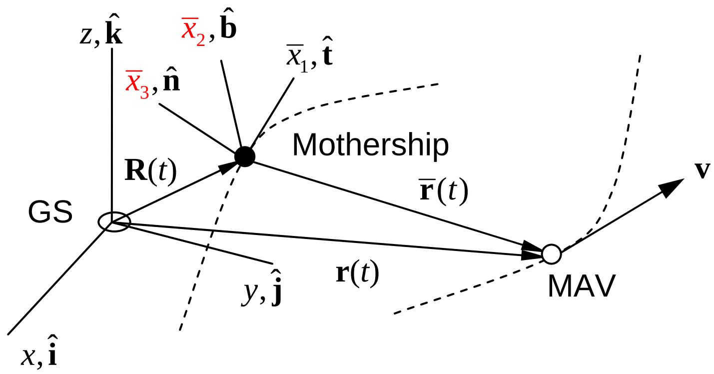
The Research and Engineering Center for Unmanned Vehicles (RECUV) is developing the concept of Cooperative Mobile Sensing Systems (CMSS). A potential scenario is shown in the following diagram. In this scenario Micro Aerial Vehicles (MAVs) are deployed from a small Unmanned Aircraft (UA) mothership into a hazardous environment. The MAVs function as “smartsondes”, i.e., sensors are integrated into the MAVs and their data are relayed back to the mothership which is sending mothership and MAV telemetry to a Ground Station (GS).

The diagram shows the path of the mothership (black dot) as seen from the GS, $\mathbf{R}(t)$. The MAV (open dot) is being tracked by the mothership, $\bar{\mathbf{r}}(t)$, using the mothership’s local Frenet frame.

We consider paths described by the Cartesian position vector $\mathbf{r}(t)$, where t is time

$$\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$$

[1] Malek-Madani, R. *Advanced Engineering Mathematics with Mathematica and MATLAB, Vol. 2*, Addison-Wesley (1998).



Tasks:

1. Plot the parameterized space trajectory $\mathbf{R}(t)$ as observed from the GS and $\bar{\mathbf{r}}(t)$ as observed from the mothership. Plot the Frenet frame unit vectors of the mothership, as observed from the GS, and the MAV as observed from the mothership, at selected points along the $\mathbf{R}(t)$ and $\bar{\mathbf{r}}(t)$ trajectories, respectively.

2. Plot the speed and magnitudes of the tangential and normal acceleration of the mothership as observed from the GS and the MAV as observed from the mothership.

3. Plot the trajectory $\mathbf{r}(t)$ of the MAV as viewed from the GS by transforming the mothership trajectory observation.

4. Plot the speed and magnitudes of the tangential and normal acceleration of the MAV by transforming the mothership observations to the GS frame.

5. Show results for the following curves with normalized time t:

   (a.) $\bar{\mathbf{r}}(t) = (\cos t, \quad \sin 2t, \quad \cos 2t), \quad t \in [0, 2\pi]$

   (b.) $\mathbf{R}(t) = (t\cos t, \quad t\sin 2t, \quad t), \quad t \in [0, 2\pi]$

6. Modify your Matlab m-file to accept position observations $\bar{\mathbf{r}}(t)$ at discrete times and repeat items 1-5 for a sample input to be provided.

Use the AIAA conference and meetings paper template at http://www.aiaa.org/content.cfm?pageid=396 and submit your report in pdf format. The document should contain a description of your approach (e.g., definitions and equations) and results. You will be graded on the organization and readability of your report, as well as the accuracy of the results. The page limit is 5. Use the file name format: lastname_initial.pdf.