ASEN 5227
AEROSPACE MATHEMATICS 1, FALL 2004

The del and Laplace Operators in Cylindrical and Spherical Coordinate Systems
(from Reddy & Rasmussen).

Cylindrical (R, ϕ, z)

$x = R\cos\phi\,,\;\; y = R\sin\phi\,,\;\; z = z$

$\hat{\mathbf{e}}_R = \cos\phi\,\hat{\mathbf{e}}_x + \sin\phi\,\hat{\mathbf{e}}_y$

$\hat{\mathbf{e}}_\phi = -\sin\phi\,\hat{\mathbf{e}}_x + \cos\phi\,\hat{\mathbf{e}}_y$

$\hat{\mathbf{e}}_z = \hat{\mathbf{e}}_z$

$$\frac{\partial\hat{\mathbf{e}}_R}{\partial\phi} = -\sin\phi\,\hat{\mathbf{e}}_x + \cos\phi\,\hat{\mathbf{e}}_y = \hat{\mathbf{e}}_\phi$$

$$\frac{\partial\hat{\mathbf{e}}_\phi}{\partial\phi} = -\cos\phi\,\hat{\mathbf{e}}_x - \sin\phi\,\hat{\mathbf{e}}_y = -\hat{\mathbf{e}}_R$$ (all other derivatives of the base vectors are zero)

$$\nabla = \hat{\mathbf{e}}_R\frac{\partial}{\partial R} + \frac{\hat{\mathbf{e}}_\phi}{R}\frac{\partial}{\partial\phi} + \hat{\mathbf{e}}_z\frac{\partial}{\partial z}$$

$$\nabla^2 F = \frac{1}{R}\left[\frac{\partial}{\partial R}\left(R\frac{\partial F}{\partial R}\right) + \frac{1}{R}\frac{\partial^2 F}{\partial\phi^2} + R\frac{\partial^2 F}{\partial z^2}\right]$$

Spherical (r, θ, ϕ)

$x = r\sin\theta\cos\phi\,,\;\; y = r\sin\theta\sin\phi\,,\;\; z = r\cos\theta$

$\hat{\mathbf{e}}_r = \sin\theta\cos\phi\,\hat{\mathbf{e}}_x + \sin\theta\sin\phi\,\hat{\mathbf{e}}_y + \cos\theta\,\hat{\mathbf{e}}_z$

$\hat{\mathbf{e}}_\theta = \cos\theta\cos\phi\,\hat{\mathbf{e}}_x + \cos\theta\sin\phi\,\hat{\mathbf{e}}_y - \sin\theta\,\hat{\mathbf{e}}_z$

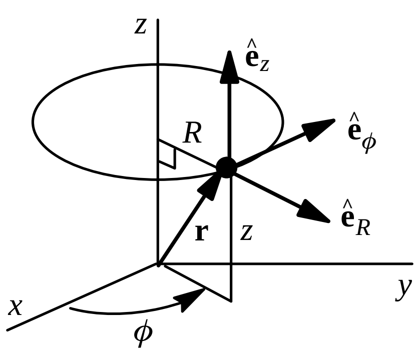
$\hat{\mathbf{e}}_\phi = -\sin\phi\,\hat{\mathbf{e}}_x + \cos\phi\,\hat{\mathbf{e}}_y$

$$\frac{\partial\hat{\mathbf{e}}_r}{\partial\theta} = \hat{\mathbf{e}}_\theta\,, \qquad \frac{\partial\hat{\mathbf{e}}_r}{\partial\phi} = \sin\theta\,\hat{\mathbf{e}}_\phi\,, \qquad \frac{\partial\hat{\mathbf{e}}_\theta}{\partial\theta} = -\hat{\mathbf{e}}_r\,, \qquad \frac{\partial\hat{\mathbf{e}}_\theta}{\partial\phi} = \cos\theta\,\hat{\mathbf{e}}_\phi$$

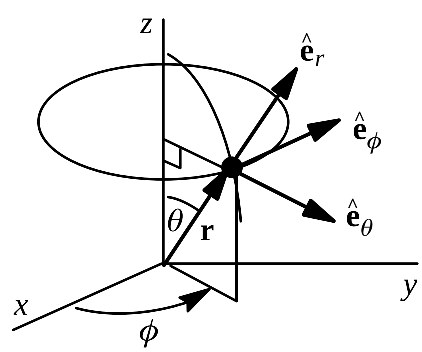
$$\frac{\partial\hat{\mathbf{e}}_\phi}{\partial\phi} = -\sin\theta\,\hat{\mathbf{e}}_r - \cos\theta\,\hat{\mathbf{e}}_\theta$$ (all other derivatives of the base vectors are zero)

$$\nabla = \hat{\mathbf{e}}_r\frac{\partial}{\partial r} + \frac{\hat{\mathbf{e}}_\theta}{r}\frac{\partial}{\partial\theta} + \frac{\hat{\mathbf{e}}_\phi}{r\sin\theta}\frac{\partial}{\partial\phi}$$

$$\nabla^2 F = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial F}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial F}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 F}{\partial\phi^2}$$



cylindrical system



spherical system