ASEN 5190 NOTES on POSITIONING ERRORS

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Introduction

A variety of error measures are used in positioning deriving from different positioning requirements.
People are generally most familiar with error measures for a scalar random variable. In navigation and positioning, two dimensional distributions are of interest for horizontal positioning. Three dimensional errors are also important, although very often the vertical direction has very different performance requirements and is specified separately.

1 - D

For a scalar measurements (x_i) we have the following

The mean value over *n* measurements is: $$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = E[x_i] \quad (1)$$

The deviation of a measurement from the mean is : $$dx_i = x_i - \bar{x} \quad (2)$$

The standard deviation is defined as: $$s = \sqrt{\frac{\sum_{i=1}^{n} dx_i^2}{n-1}} \quad (3)$$

The variance is σ^2 .

The root mean square value is : $$RMS = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}} \quad (4)$$

For a scalar random variable (or measurement) with a Normal (Gaussian) distribution, the probability of being within 1-σ of the mean is 68.3%.

2 - D

For a two dimensional measurement, say x_I and y_I we have,

The mean value over *n* measurements is: $$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \quad (5)$$

The deviation of a measurement from the mean is : $$dx_i = x_i - \bar{x} \qquad dy_i = y_i - \bar{y} \quad (6)$$

The covariance matrix is: $$P = \begin{bmatrix} s_x^2 & \sum_{i=1}^{n} \frac{dx_i dy_i}{n-1} \\ \sum_{i=1}^{n} \frac{dx_i dy_i}{n-1} & s_y^2 \end{bmatrix} \quad (7)$$

The covariance matrix defines the error ellipse. The eigenvalues of P are the squares of the semimajor and semiminor axes of the ellipse (σ_1^2 and σ_2^2) and the eigenvectors show the orientation of the error ellipse.

DRMS - Distance Root Mean Square 2-D

DRMS = $[\sigma_1^2 + \sigma_2^2]^{1/2}$ = Radial Error, Mean Square Position Error, or RSS (8)

Probability of being inside DRMS circle :

For $\sigma_1 = \sigma_2$ probability = 63%

For $\sigma_1 = 10\sigma_2$ probability = 68%

2DRMS - Two definitions

- 2 x DRMS : Probability between 95.4% and 98%
  (US Federal Radionavigation Plan definition)

- 2-D RMS : Same as DRMS (63-68%) (NATO's Standardization Agreement)

CEP - Circular Error Probable is the radius of a circle inside which 50% of the points fall.

How do you compute this? From the data, find the <u>median</u> radius.

How does it compare to σ_1 and σ_2?

For a normal distribution

For $\sigma_1 = \sigma_2$ CEP = 1.18 σ

Approximate : CEP = 0.59 $(\sigma_1 + \sigma_2) \pm 3\%$ for $\sigma_2 / 3 < \sigma_1 < 3\,\sigma_2$ (9)

95% circle is CEP x 2.08 = 2 x DRMS

99% circle is CEP x 2.58

3 - D

Error ellipsoid - Based on covariance matrix, same as 2-D

Probability of being inside the 1-σ ellipsoid is 20%.

MRSE - Mean Radial Spherical Error

MRSE = $[\sigma_1^2 + \sigma_2^2 + \sigma_3^2]^{1/2}$

Probability of being inside MRSE sphere : 61%.

SEP - Spherical Error Probable is the radius of sphere inside which 50% of the points fall.

Approximate : SEP = 0.59 $(\sigma_1 + \sigma_2 + \sigma_3)$

Lab 1 Comments and Questions

After reviewing the example below, think about how this relates to your data for Lab#1.

- Do you think your errors are Gaussian?
- What percentage of the points lie inside the 1-σ error ellipse?
- If the error distribution appears skewed in a particular direction, what do you think is causing it?
- How do your statistics compare to the estimates of GPS accuracy given in the book?

References:

1. *Aerospace Avionics Systems - A Modern Synthesis*, G. M. Siouris, Appendix A, 1993.