Design Problem 3

1 Introduction

The purpose of this design problem is to familiarize yourself with the optimization of a design for a combined aerodynamics and structural performance. From a purely aerodynamic point of view, increasing the wing size will always give you a better lift to drag ratio. However the higher the wing area, the higher the structural weight, ultimately increasing your total drag. The structural weight will increase because the size of the wing itself is larger, but also because as the area increases, the gust sensitivity increases. This will also result in a heavier fuselage weight.

In this problem we will perform the wing sizing to minimize the total drag when the effect of the weight of the wing has been taken into account.

2 Data

The following characteristics have already been chose and/or determined:

- Fuselage length $38.28m$. Fuselage diameter $3.15m$
- Aspect ratio of the wing: $AR = 8.7$
- Optimum Cruise Mach number: $M = 0.79$
- Maximum Cruise Mach number: $M_C = 0.84$
- Dive Mach number: $M_D = 0.90$
- The wing is trapezoidal with a sweep of $\Lambda_{\frac{c}{4}} = 30^o$ and a a taper ratio of $\sigma = 0.21$.
- Average thickness ration $(\frac{t}{c}) = 11.5\%$
- In flight maximum lift coefficient $C_{L_{max}} = 1.21$
- Specific fuel consumption of the engines in cruise: $0.58\ lb/(lb.h)$
- Specific loitering fuel consumption: $0.51\ lb/(lb.h)$
- The distribution of the wing weight along the span is proportional to the chord length at every station.
- The fuel weight is distributed along the span proportional to the square of the chord length at every station.
- Fuel reservoirs start 2.0m from the centerline and extend to 80% of the span.
- Maximum Payload mass of $15,000kg$.

- Maximum Fuel mass of $18,000kg$.
- Maximum Paylod plus Fuel mass of $29,000kg$.

3 Assignment

In this design assignment you will have to perform the following tasks:

1. Optimize the wing size for best performance.
2. Compute the V-n diagrams for the following cases:
   (a) Full payload and maximum allowable fuel.
   (b) Full payload, zero fuel.
   (c) Zero payload, $800kg$ of crew and equipment and maximum fuel.

   Diagrams should be obtained for altitudes of 20,000 ft and 35,000 ft.
3. Estimate the critical (maximum) wing bending moment at the root. Note for what fuel, payload weight and flight conditions this critical case is attained. (The span-wise lift distribution can be assumed to be elliptic) Attach a chart of the moment as a function of the spanwise position along the quarter chord line.
4. Derive the Payload-range characteristic of the airplane.

Some of the work is to actually derive the appropriate computations of weights and total drag. Your report should clearly show what are the equations you have obtained and that you are using in your programs.

4 Optimization of the wing area

The first part of this design is to optimize the wing area. For this purpose, we are looking for the wing area that will minimize the total drag in flight, when the total mass of cargo, passengers and fuel transported is $23,000kg$. In what follows, we will denote the structural weight of the wing by W_w. Then, the weight of the wing will be given by the formula from page 391:

$$\frac{W_w}{S_w} = 4.25 + 1.65 I_w \quad (lb/ft^2)$$

The empty weight, not counting the wing is

$$W_r = 21500 + N_{ULT} * 4200 \quad (lb)$$

The wing and fuselage will be designed so that it can take the Ultimate load factor N_{ULT}, which will be the highest of:

- $N = 2.5$ maneuvering loading.
- The rough air gust load with a weight of $W_w + W_r + 2000lb$ at 20,000ft.
- The maximum cruise gust with a weight of $W_w + W_r + 2000lb$ at 20,000ft.

To perform this optimization, you will use matlab programming capabilities. One approach to solving this problem is as follow:

- Write a matlab function $Estweight(S, W_0, W_p, W_f)$ that computes the estimated weight for the wing and the rest of the structure, when the following is given:
  1. S, the total wing area
  2. $W_0 = W_w + W_r$, the estimated weight for the empty aircraft structure (wing, fuselage, equipments and engines)
  3. W_p, the maximum payload weight
  4. W_f, the maximum fuel weight with the above payload
- Write a matlab code to solves $Est_weight(S, W_0, W_p, W_f) = W_0$ for the variable W_0. We will call the the solution of this equation for the given payload and fuel weight $W_0^*(S)$.
- Write a function $Drag(S, W)$ that computes the total drag of the aircraft at optimal $(\frac{L}{D})$
- Plot $Drag(S, W_0^*(S) + W_p + \frac{1}{2}W_f)$ as a function of the wing area

The last step produces a plot that will have a minimum. Chose a wing area near that minimum for your design.

5 Skin Friction

For the skin friction coefficient, use the standard aircraft roughness friction:

$$C_f = 0.55015(log_{10}Re)^{-2.58}$$

Base your Reynolds computation on a flight altitude of $30,000ft$. Assume the wing and fuselage create 84% of the total parasitic drag. You can assume that the wing is exposed from $1.25m$ from centerline onward.

6 Load and bending moment computations

An easy way to perform the computation of bending moment is to use a spreadsheet program. In the spreadsheet, create the following rows:

1. the distance of a point on the wing from the root. (along the quarter chord line)

2. the lift per unit length. This is written as a function of the previous row. For example: $B8 = l_0 * sqrt(1 - (2 * A8/b)^2)$. Where l_0 should be replaced by the appropriate value so that the total lift equals the total weight under loading conditions and b should be replaced by the proper span measured along the quarter chord line.

3. the structural weight per unit length at each station for the load factor

4. the fuel weight per unit length at each station at the current load factor

5. the total force acting between station i and station $i + 1$: $E8 = (A9 - A8) * ((B8 + B9)/2 - (C8 + C9)/2 - (D8 + D9)/2)$

6. the shear force in the z axis from the tip to the current station. Note that this can be achieved by a recurrence formula from the station just outboard of the present one: $F8 = F9 + E8$

7. the moment at the current station. Again a recurrence formula based on the moment at the next station and the shear force at the middle point between the current station and the next will give you the right answer.